

# Bad Beta and Good Beta Revisited: Rational and Irrational Expectations

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## Abstract

Campbell, Polk and Vuolteenaho (2010) determine the source of systematic risks in asset prices by assuming that the cash flow news is driven by fundamentals whereas discount rate news is sentiment driven. This study empirically evaluates their assumptions by constructing a four-beta model that disentangles the cash flow and discount rate betas of Campbell and Vuolteenaho (2004) into rational and irrational components. The empirical results do not support their assumptions in that the stock returns respond significantly to the shocks in the irrationally expected cash flow and rational discount rate. Comparing the asset pricing performance of our four-beta model against alternative asset pricing models reveals that our model has a better model fit with lower pricing error. The documented negative (positive) risk premia of irrational (rational) betas implies that investors are willing to pay a price (require a risk premium) for stocks that are sensitive to the irrational risk factors (rational risk factors).

*Keywords:* Rational expectations; Irrational expectations; Cash flow; Discount rate; Asset pricing

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## 1. Introduction

The most widely used asset pricing theory proposed by Sharpe (1964) and Lintner (1965) – capital asset pricing model (CAPM) – argue that the cross-section of stock returns is described by a single systematic risk, measured by the market beta. Campbell and Vuolteenaho (2004, henceforth CV) improve the explanatory power of CAPM on the cross-section of stock returns by disentangling the CAPM beta into cash flow and discount rate betas, stemming from the key concept of the asset pricing theory that asset price is derived by discounting the expected future cash flow. The CV’s two-beta model does not distinguish between the effects of rational and irrational expectations of cash flow and discount rate on the stock prices. Irrational investors who trade based on their sentiment tend to form irrational expectations about future cash flows and returns, affecting stock prices and returns. Hence, we revisit and extend their two-beta model to a four-beta model that explicitly acknowledges the role of irrationally expected future cash flow and discount rate<sup>2</sup> on the stock prices given that previous literature have documented the role of irrational expectations on the stock price.

Traditional finance theory is built on the assumption that investors discount the rationally expected cash flow at an appropriate discount rate. This gives rise to the return decomposition framework of the Campbell and Shiller (1988a) and Campbell (1991), where unexpected stock market returns constitute of the market cash flow news ( $N_{CF}$ ) and the market discount rate news ( $N_{DR}$ )<sup>3</sup>. Build upon this framework, CV (2004) decompose the CAPM beta into ‘good’ discount-rate beta ( $\beta_{i,DR}$ ) that measures the response of stock to the  $N_{DR}$ , and ‘bad’ cash-flow beta ( $\beta_{i,CF}$ ) that measures the reaction of stock to the  $N_{CF}$ . The terminology of ‘bad’ and ‘good’ beta is used since, as explained by authors, a long-term risk averse investor would require a greater premium on stocks that are more sensitive to the  $N_{CF}$ , which causes a permanent and irreversible effect, than the stocks that are more sensitive to the  $N_{DR}$ , which effect tend to be transitory.

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<sup>2</sup> Investor irrationality is a broad term in that it is a reflection of different types of investor psychology. Thus, the whole magnitude of investor irrationality in the stock market is unknown. Since investor sentiment has been well recognised as a proxy to investor irrationality, the variations of sentiment-induced expectation is termed as irrational expectations in the remainder of this paper.

<sup>3</sup>  $N_{CF}$  ( $N_{DR}$ ) is the changes in expectation about the future cash flow (future stock returns).

Whilst their two-beta model has yielded an impressive explanation for the higher average returns of value and small stocks in the post-1963 period, their model is silent on differentiating the irrational expectation from the rational expectation of future cash flow and discount rates. Unlike CV (2004), Campbell, Polk and Vuolteenaho (2010, CPV hereafter) distinguish the fundamental and sentiment view based on the cash-flow and discount rate movement of firms with the market news. Specifically, the systematic risks<sup>4</sup> of stocks are said to be driven by fundamental factor if the co-movements of the stock returns with market news are caused by their cash-flow movements. Otherwise, the investor sentiment is said to play an important role in explaining the systematic risk of the stocks if the discount rate of stocks mainly drives the systematic risks. They found that the systematic risks of value and growth stocks are mainly driven by their cash flow news, and hence claim that the systematic risks of growth stocks are driven by their fundamentals instead of sentiment as claimed in previous studies<sup>5</sup>.

Other studies, such as Da and Warachka (2009) and Koubouros, Malliaropulos and Panopoulou (2010), although, do not aim to distinguish between fundamental and sentiment view, they tend to perceive that the cash flow risk is link to the fundamental. Chen and Zhao (2009) also mention that  $N_{CF}$  is link to fundamental factors and the  $N_{DR}$  could be due to a change in sentiment or risk aversion.

Despite these assumptions, the changes in expectations about the future cash flow and discount rate can, however, reflects both rational and irrational expectations of investors, and stock prices react to both the rational and irrational components in each shock. Indeed, as defined in Baker and Wurgler (2007), investor sentiment is the expectation about future cash flow and risk that is not justifiable by fundamental information. Hence, a change in the investor sentiment could reflects a change in the irrational expectations of future cash flow

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<sup>4</sup> The systematic risk measures adopted in CPV (2010) are the bad and good betas of CV (2004), which claim that value (growth) stocks have higher bad cash flow (good discount rate) beta. Nevertheless, such a pattern of bad and good betas is not documented for value and growth stocks in this study, which has a different sample period, and therefore allowing for both rational and irrational expectation to play a role in both cash flow and discount rate channels would be more appropriate.

<sup>5</sup> Since assets' cash flow news (discount rate news) is correlated to the markets' cash flow news (discount rates news) (Pettit and Westerfield, 1972), the claim of CPV (2010) made at the stock-level also implicitly implies that the changes in the market-wide cash flow expectations is driven by fundamental factor; changes in market discount rate is driven by investor sentiment.

and/ or returns, which would then lead to an unexpected move in the stock price. As shown in the simple model by Brown and Cliff (2005), the stock price is the weighted average of prices formed based on rational and irrational expectations of future cash flow and future return<sup>6</sup>. Therefore, unexpected return could be ensued from the revision in both the rational and irrational expectations of future cash flow and/ or discount rates.

Empirically, investors forming irrational expectation about future cash flow is well documented in the literature (*e.g.* Barberis, Shleifer and Vishny, 1998; Cooper, Gullen and Schill, 2008; Engelberg, Mclean and Pontiff, 2018; Hribar and McInnis, 2012; Lakonishok, Shleifer and Vishny, 1994 (LSV henceforth); Piotroski and So, 2012). Stock prices could be greatly affected if investors form systematic expectation errors of future cash flow. Lamont and Thaler (2003, p.201) question that “During the Nasdaq bubble of the late 1990s, approximately \$7 trillion of wealth was created and then destroyed. Was this a rational process of forecasting the future cash flows of new technology or an investing frenzy based on mob psychology?”. Indeed, past studies mention that investors’ irrational expectations of earnings growth leads to the formation of Dot-com bubble (Ofek and Richardson, 2002<sup>7</sup>) and the overpriced of internet-based IPOs (Loughran and Ritter, 1995; Ritter 1991). Therefore, irrationally expected cash flow should not be completely ruled out from an asset pricing model even though CPV (2010) claim that investor sentiment can only have an indirect effect on the cash flow. Furthermore, investor sentiment is highly persistent, current expectations about future cash flow could be affected by previous sentiment that lasts for periods, and hence it is hard to claim that  $N_{CF}$  links solely to the fundamental factors.

As opposed to the sentiment view of CPV (2010) on the expected discount rate, other studies show that discount rate news could have rational explanations. Changes in the expected discount rate could reflect the compensations for the time-varying risk (*e.g.* Bansal and Yaron, 2004; Bollerslev, Tauchen and Zhou, 2009) and/ or the risk aversion (*e.g.* Campbell and Cochrane, 1999; Cochrane, 2011). In his presidential address, Cochrane (2011)

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<sup>6</sup> Such pricing model, *i.e.* asset prices are determined as the weighted average of expected payoffs formed by heterogeneous investors, can be traced back to Diether, Malloy and Scherbina (2002), Lintner (1969) and Miller (1977).

<sup>7</sup> They reported that 6% of total market capitalization in the US stock market is represented by internet-based stocks even though these stocks have negative earnings, which are priced in the market, before the burst of Dot-com bubble. Similar evidence is presented by Schultz and Zaman (2001, p.354).

argue that discounting the future payoffs at a risk-free rate with distorted probability is simply equivalent to discounting the future payoffs at a different discount rate. Having said so, behavioural explanations have been proposed to explain the variation in expected returns (e.g. Barberis, Greenwood, Jin and Shleifer, 2015; Greenwood and Shleifer, 2014; Cassella and Gulen, 2018). As Cohen, Gompers and Vuolteenaho (2002) mentioned that discount rate news can be treated as the mispricing news as well as a change in the firm's risk. Hence, it is important to account for both rational and irrational expectations of future returns in an asset pricing model.

To evaluate the assumptions made in previous literature, we construct a four-beta model by decomposing the cash flow and discount rate betas into rational and irrational components. Each beta in our model measures the covariances of asset returns with one of the news series – irrational cash flow news ( $N_{CF}^{IR}$ ), rational cash flow news ( $N_{CF}^R$ ), irrational discount rate news ( $N_{DR}^{IR}$ ), and rational discount rate news ( $N_{DR}^R$ ). If the claims made by CPV (2010) are true, then the covariances of asset's returns with the irrational cash flow news and the rational discount rate news would not be significantly different from zero. That is to say, asset prices will not react to, for instance, the changes in the irrational expectations of market cash flow, which constituted of the irrational cash flow news from individual stocks, if  $N_{CF}$  is mainly driven by fundamental factors. If both rational and irrational expectations significantly affect the stock prices, is the covariance of stock returns with the shocks in each expectation is significantly priced across different stocks? Hence, we investigate whether each beta component in our model is a systematic risk that is priced at the cross-section level.

Methodologically, we first disentangle the unexpected returns into cash flow news and discount rate news by using the Vector Autoregression (VAR) approach following Campbell (1991) and CV (2004). However, unlike CV (2004) who assume the true VAR parameters are constant over the full sample period, we allow the parameter estimate of each state variable to vary over time since the literature argued that parameter instability is accountable for the time-varying predictive strengths of the state variables on the future stock market returns (see Lettau and Nieuwerburgh, 2008; Henkel, Martin and Nardari, 2011; Pesaran and Timmermann, 2002). By doing so, the new series could be more precisely reflect the shocks in the stock returns over time. We term this approach as the time-varying VAR

(TV-VAR)<sup>8</sup>. To investigate the pricing of the four betas, we perform the Fama-Macbeth (1973, henceforth FMB) regression to obtain the risk premium of each risk factor.

Asset pricing theory states that only the risk factors that systematically affect all stocks are priced. Whilst different fundamental risks have been considered and priced in the rational asset pricing models (*e.g.* Bansal and Yaron, 2004; Campbell and Cochrane, 2000; Engle and Mistry, 2014; Jagannathan and Wang, 1996), behavioural studies also found that trading from irrational investors, and so their sentiment and irrational expectations, can generate systematic risk (see Barber, Odean and Zhu, 2009; Lee, Jiang and Indro, 2002; Piotroski and So, 2012), which are priced (*e.g.* Liang 2018; Piotroski and So, 2012; Shefrin, 2008; 2015). Therefore, the risk premium in the market could constituted both rational and irrational premium. Since Liang (2018) and Fong and Toh (2014) report a negative risk premium for sentiment factor in the cross-section of stock returns, we conjecture that the irrational risk factors in this study would command a negative risk premium<sup>9</sup>.

Empirically, our four-beta model confirms that stocks are not immune to the variations in the irrational cash flow expectations and rational discount rate expectations since the irrational cash flow beta and rational discount rate beta estimates are significant across different portfolios. Hence, the null hypothesis that the covariances of asset's returns with the irrational cash flow news and the rational discount rate news would not be significantly different from zero is rejected by our model. The results from the full sample VAR estimation further supports our findings obtained from the TV-VAR estimation, where only the irrational cash flow news and rational discount rate news are found to be significantly affect the stock price under different estimation frameworks. As for the asset pricing test, we find that the four-beta model improves the explanatory power of the CAPM and CV's two-beta model in describing the cross-sectional variation of average excess returns since our model delivers a positive  $R^2$  statistic and a lower pricing error. In line with our prediction, the irrational betas are significantly and consistently priced in the cross-section of stock returns and demand a negative risk premium. On the other hand, covariances of asset's

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<sup>8</sup> As Chen and Zhao (2009) show that the news series estimated from different sample period alter the conclusion of beta trend documented in CV (2004), which can be seen also in Section 5.2, hence we adopt a time-varying VAR approach to estimate the news series. Although our baseline results come from the TV-VAR, we also provide the results derived from VAR as a comparison.

<sup>9</sup> The rationale of the negative risk premium associated with the irrational betas are discussed in Section 6.5.

returns with the news of rational expectations earns a positive risk premium. However, the risk premium estimates associated with the rational cash flow and discount rate betas have lower magnitude in the absolute term.

A popular issue documented in the asset pricing literature is that the market beta is varying over time (see Jagannathan and Wang, 1996; Merton, 1973), and hence time-varying beta computed using rolling window approach is widely adopted (*e.g.* Adrian and Franzoni, 2009; Botshekan, Kraeussl and Lucas, 2012; Petkova and Zhang, 2005). In view of the fact that the beta is non-constant, we also perform the sub-sample analysis in order to investigate if the findings on the beta estimates produced under the TV-VAR framework change across different sub-samples, and how these changes affect the pricing of each beta risk. In particular, we employ the structural break test to identify the structural shifts in the four betas<sup>10</sup>.

The sub-sample analysis reveals that the changes in the irrationally expected cash flow significantly affect most of the assets' returns in the second sub-sample period, but not in the first sub-sample period. In contrast, the rational discount rate betas across all portfolios remain highly significant in both sub-sample periods. These findings are again calling into question the assumptions of CPV (2010). Meanwhile, the positive sign of irrational cash flow and rational discount rate beta estimates remain unchanged across both periods. As for the pricing of risk, consistent with the full sample results, both irrational cash flow and irrational discount rate betas are significantly priced and investors are willing to pay an insurance for the irrational beta risks across sub-sample periods.

This study contributes to the literature in several aspects. First, we evaluate the assumptions that commonly applied in the literature, especially the one made in CPV (2010). Often, cash flow news is claimed to be driven by fundamental factor, but none of the studies have split the cash flow news into rational and irrational components in order to examine if the fundamental factor is the only driver for the cash flow news. This study provides the first examination on the assumption made with respect to the cash flow news. Similarly, CPV

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<sup>10</sup> To be able to estimate the risk premia associated with the four betas, we need to have a consistent break point for each of the four betas. Although we could have incorporated multiple break points in this study, but the test for multiple break reveals inconsistent break points across different betas and this complicates the analysis. Hence, for simplicity, we adopt a single break test to identify the main structural break in our sample.

(2010) made a definite claim on the discount rate news that it is driven mainly by the sentiment. Again, no study has tested on whether discount rate news is truly driven by investor sentiment only, or rational expectation does play a role as well. Thus, our study fills this gap, validating the assumptions of cash flow and discount rate news by using the four-beta model constructed in this study.

Second, to the best of our knowledge, no one has developed the four-beta model that decomposes the cash flow and discount rate betas into rational and irrational components, in order to examine the pricing of the those betas in the cross-section of stock returns. CV (2004) mention that their model remains important in understanding how a long-term risk-averse investor prices the cash flow and discount rate risks even though investor irrationality could have affected the stock prices. However, investor irrationality has not been given a credit explicitly in their model. Therefore, extending their two-beta model to a four-beta model that accounts for both rational and irrational expectations could further enhance our understanding towards the pricing properties of rational and irrational risks in both cash flow and discount rate channels.

Last but not least, we add to the literature of behavioural finance in that this study provides a deeper understanding on the economic source underlying the sentiment-return relation. Although Huang et al. (2015) also examine the underlying source of sentiment-return relation, their investigation focused on the one-period model. Our study hence complements to their work in that we consider the multiperiod model, which is modelled through the VAR specification, on the ground that we are evaluating the behavior of long-lasting securities, and that the sentiment exerts stronger predictive power on the long-horizon stock returns (see Brown and Cliff, 2005; and Yu and Yuan, 2011). Furthermore, studies that examined the effect of irrational expectation on only either the future cash flow or the expected returns, separately. We fill the gap by integrating irrational expectations in both cash flow and discount rate channels into one model since the valuation of an asset could be affected by the expectations errors about the future cash flow and discount rates concurrently. The newly constructed model permits a better comparison on the relative importance of the sentiment-induced irrational cash flow and discount rate expectations on the asset's returns.



This paper is organized as follows. Section 2 presents the framework of return decomposition. Section 3 discusses the empirical methodology employed in this study, which includes the approaches used to decompose stock market returns, the computation of the four-beta model, and the pricing of the four betas. Section 4 presents the data and descriptive statistics of data. Section 5 presents the results and discussion. Section 6 present the empirical application of the four-beta model on a set of equity anomalies. Last section concludes.

## 2. Return decomposition

Based on the present value concept, stock prices change because of a change in the expected cash flow and/ or discount rate. An increase in the expected future cash flow will lead to an increase in the stock prices; an increase in the discount rate will cause a drop in stock prices. This simple concept leads to the development of the return decomposition framework introduced by Campbell and Shiller (1988a) and Campbell (1991). The framework starts with the log-linear approximation of the present value approach (Campbell and Shiller, 1988a), where the next-period stock returns approximates to the log-linear return around the mean of log dividend-price ratio, using the first-order Taylor approximation, can be expressed as  $r_{t+1} \approx k + \rho p_{t+1} + (1-\rho)p_t - d_{t+1} - p_t$ . The lowercase letters  $r_t$ ,  $p_t$  and  $d_t$  denote the log transformed stock returns ( $R_t$ ), stock price ( $P_t$ ) and dividend ( $D_t$ ), respectively.  $k$  is a constant term expressed as  $k = -\log \rho - (1-\rho)\log(1/\rho - 1)$  and the discounting coefficient,  $\rho$ , is assumed to be a constant such that  $\rho = P/(P+D)$ .

Iterating the one-period log-linear return approximation forward with  $\lim_{j \rightarrow \infty} \rho^j (d_{t+j} - p_{t+j}) = 0$  yields the following linearized present value identity, which is an ex-ante measure with an expectation notation:

$$p_t - d_t = \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j [\Delta d_{t+1+j} - r_{t+1+j}] \quad (1)$$

where  $E_t$  denotes the expectations made at time  $t$ . This model implies that the increase in the expectation of future log dividend growth,  $\Delta d_{t+1+j}$ , and/ or a drop in the expectation of future stock returns,  $r_{t+1+j}$ , will produce a high log price-dividend ratio,  $p_t - d_t$ . The assumption of

$\lim_{j \rightarrow \infty} \rho^j (d_{t+j} - p_{t+j}) = 0$  implies that the mean reverting condition holds for the terminal value of log price-dividend ratio.

Instead of employing the above present value identity to imply the forecasts of stock returns, Campbell (1991) explicitly forecast the stock returns. He decomposed the stock returns into the expected cash flow and the expected return components, and derive the equation for the unexpected stock returns as follow:

$$r_{t+1} - E_t(r_{t+1}) = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1,j} \quad (2)$$

$$r_{t+1} - E_t(r_{t+1}) = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} - \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1,j} \quad (3)$$

where  $E_t$  represents the expectations made at time  $t$ . The unexpected stock returns,  $r_{t+1} - E_t(r_{t+1})$ , at time  $t+1$  is simply the combination of the change in the expectations,  $E_{t+1} - E_t$ , of cash flow and discount rate at time  $t+1$ . The shocks in these return components are defined as cash flow news,  $N_{CF}$ , and discount rate news,  $N_{DR}$ .

$$N_{CF,t+1} = \Delta E_{t+1} \sum_{j=0}^{\infty} \rho^j \Delta d_{t+1+j} \quad (4)$$

$$N_{DR,t+1} = -\Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1,j} \quad (5)$$

The above equations indicate that a decrease in the unexpected return is a result of a decrease in the current and expected future cash flow and/ or an increase in the discount rates, and vice versa. The negative relationship between unexpected return and discount rate news is intuitive. A higher future stock returns can only be realized from a lower current stock price (*i.e.* currently suffer from a loss), assuming the dividend growth holds constant. An investor, who is considering adding an additional stock into a well-diversified portfolio, will need to make the decision based on the comovement of that particular stock with the stock market news, which are  $N_{CF}$  and  $N_{DR}$ , and this leads to the construction of two-beta model in

CV (2004). The empirical estimation of  $N_{CF}$  and  $N_{DR}$  as well as the extension of the two-beta model into four-beta model are presented in the next section.

### 3. Empirical methodology

This section presents three different approaches used to decompose the stock market returns in  $N_{CF}$  and  $N_{DR}$ . The decomposition of the two-beta model into rational and irrational channels in order to yield a four-beta model is then explained in detailed.

#### 3.1 Return decomposition approaches

##### 3.1.1. VAR Approach

To operationalize equation (2), Campbell and Shiller (1988a) and Campbell (1991) propose the use of vector autoregression (VAR) model in decomposing the stock market returns. The main idea of this approach is to extrapolate the short run forecast of the stock market returns into the long run forecasts since the data of the state variables over the infinite period (or the long horizon) is hard to obtain. First, we assume that the stock market returns is generated by the first-order VAR model following CV (2004).

$$\mathbf{z}_{t+1} = \mathbf{a} + \mathbf{\Gamma}\mathbf{z}_t + \mathbf{u}_{t+1} \quad (6)$$

with the stock market returns be the first element in a  $m \times 1$  state vector,  $\mathbf{z}_{t+1}$ , and other state variables constitute any of variables that are known to predict the stock market returns.  $\mathbf{a}$  is a  $m \times 1$  vector of constant,  $\mathbf{\Gamma}$  is a  $m \times m$  matrix of constant slope coefficients and  $\mathbf{u}_{t+1}$  is the  $m \times 1$  vector of random shocks. Although studies by Cohen et al. (2002), CPV (2010), and Khimich (2017) employ stock returns as the first element in  $z_{t+1}$ , this study opts for stock market returns for two reasons. First, we follow closely the procedure of CV (2004) who decompose the stock market returns instead of individual stock returns. Second, the effect of investor sentiment is pervasive in the stock market. This enables us to estimate the irrational component of the stock market news.

Whilst the real stock market return,  $r_{M,t+1}^e$ , can be retrieved from the vector  $\mathbf{z}_{t+1}$  as  $r_{M,t+1}^e = \mathbf{e}\mathbf{1}'\mathbf{z}_{t+1}$ , where  $\mathbf{e}\mathbf{1}' = [\mathbf{1}, \mathbf{0}, \dots, \mathbf{0}]$ , the one-period unexpected stock market returns can be computed as  $r_{M,t+1}^e - E(r_{M,t+1}^e) = \mathbf{e}\mathbf{1}'\mathbf{u}_{t+1}$ . Given that the simple multi-period forecasts of future stock market returns can be generated from the first-order VAR as  $E_t r_{M,t+1+j}^e = \mathbf{e}\mathbf{1}'\mathbf{\Gamma}^{j+1}\mathbf{z}_t$ , the discount rate news, which is the change in the discounted sum of the future expected returns over the long-run, can be estimated as:

$$N_{DR,t+1} = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1+j}^e \quad (7)$$

$$N_{DR,t+1} = \mathbf{e}\mathbf{1}' \sum_{j=1}^{\infty} \rho^j \mathbf{\Gamma}^j \mathbf{u}_{t+1} = \mathbf{e}\mathbf{1}' \boldsymbol{\lambda} \mathbf{u}_{t+1} \quad (8)$$

Where  $\boldsymbol{\lambda} = \rho \mathbf{\Gamma} (\mathbf{I} - \rho \mathbf{\Gamma})^{-1}$ ,  $\mathbf{e}\mathbf{1}' = [\mathbf{1}, \mathbf{0}, \dots, \mathbf{0}]$ ,  $\mathbf{\Gamma}$  is the point estimates of the VAR matrix, the discounting coefficient,  $\rho$ , is set at  $0.95^{1/12}$  (see CV, 2004)<sup>11</sup> and  $\mathbf{u}_{t+1}$  is the error terms of the VAR system. The cash-flow news,  $N_{CF}$ , is simply the difference between the total unexpected stock market return and the  $N_{DR}$ , and can be computed as  $N_{CF,t+1} = (\mathbf{e}\mathbf{1}' + \mathbf{e}\mathbf{1}'\boldsymbol{\lambda})\mathbf{u}_{t+1}$ .

This study accounts explicitly for the irrational expectations of future cash flow and discount rate. Unlike Lof (2015) which allows for a non-zero limiting value of dividend price ratio in the short term, this study follows Campbell and Shiller (1988a) and Campbell (1991) that the terminal condition of dividend price ratio is non-explosive, which is the assumption of equation (1). Even though we account for the irrational expectations of the future cash flow and discount rate, the irrational expectations do not always lead to the occurrence of a bubble, rather it does have an impact on the stock prices even during the normal period. For each expectation, we should therefore decompose it into rational and irrational components for the beta computation.

The return decomposition framework of Campbell and Shiller (1988a) and Campbell (1991) utilizes the financial theory in forming the expected returns since the investors' expectations are not directly observable and are extracted from the dynamic relations between the stock market returns and its predictors. Therefore, the expectations formed are rational if

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<sup>11</sup> Chen and Zhao (2009) and CV (2004) find that their results are robust to the use of different discounting coefficients,  $\rho$ . Hence, this study follows the norm in the beta decomposition literature.

and only if the VAR follows a true data generating process. However, Lof (2015) shows that the VAR does not account for all expectations due from different agents and that the prices produced by irrational contrarian model<sup>12</sup> is closer to the direction of true prices as compared to rational speculator model. Despite Lof (2015) modifying the return decomposition of Campbell and Shiller (1988a) to allow for a rational bubble by relaxing the assumption of non-explosive terminal condition of dividend price ratio, their short term strategies, however, still based on the rational expectation of speculators. For the irrational contrarian model, they simply take the opposite direction of the expected returns formed by rational speculators. This procedure, however, does not rule out the possibility that some of the contrarian investors are rational. Therefore, it is important to explicitly consider the effect of investor sentiment in forming the irrational expectations.

### *3.1.2. Time-varying VAR (TV-VAR)*

The constant parameter estimates retrieved from the VAR may not truly reflect or capture the expectations formed by investors through the time. As shown in Neely and Weller (2000), estimating a VAR on a rolling window basis greatly improves the forecasting performance, implying the parameter instability of VAR process. In view of this, this study modifies slightly the constant VAR procedure, constructing the news series from the TV-VAR approach. Specifically, the VAR parameters and the news series are estimated on a rolling window basis with a window size of 72 months<sup>13</sup>. Since the estimation window which produces the news series that best describe the evolution of stock market returns is unknown, the news series are averaged across different windows at each point in time in order to obtain a single series of cash flow and discount rate news. Then, the sentiment-induced irrational component and the rational component from each news series are extracted, producing the four news series, which are the irrational cash flow news, the rational cash flow news, the irrational discount rate news and the rational discount rate news.

There are pros and cons associated with the constant VAR and TV-VAR specifications. The advantage of the constant VAR specification is that retrieving the news series from the full sample is less subject to the small sample bias. However, applying

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<sup>12</sup> Contrarian is an investing strategy where the investors trade against the market trend.

<sup>13</sup> The window size is equivalent to a business cycle according to the NBER.

constant weights to state variables may not capture optimally the variation in the expected returns, which is well depicted in the Figure 1 that plots the estimated coefficients (panel A) of the return predictive regression from the TV-VAR model associated with the  $p$ -value (panel B) for each state variable on a rolling window basis. Panel A clearly shows that the estimated coefficient of each state variables is changing over time. Moreover, their predictive strengths are not constant through time as depicted in panel B, where each state variable predicts significantly the future stock market returns at certain periods but not the others. Hence, accurately modelling the expected returns over time is important in retrieving the real unexpected returns that will contribute to the construction of the news series. On the other hand, TV-VAR relaxes this restriction, allowing the contribution or weight of each state variable in the VAR specification to change through time. Nevertheless, the potential small sample bias faced by TV-VAR could introduce an upward or a downward bias on the estimates as compared to the constant VAR estimates<sup>14</sup>. As such, it is a trade-off between “correctness” and small sample bias. Employing a window length of less than 6 years (a full business cycle) could potentially induce small sample bias since all state variables are highly autocorrelated. Meanwhile, a longer estimation window length of 15 years has been tested and this longer window length has been found to be less optimal in capturing the variation in expected returns (the adjusted- $R^2$  for the return regression is only 2.1% as compared to that of in Table 2). Hence, the window length is chosen to mitigate the small sample bias, yet uncover the temporal variation in the expected returns. If both constant VAR and TV-VAR produce commonality in the betas estimates, the results would be more convincing and reliable. Thus, the beta estimates from both approaches are presented even though the baseline results are derived from the TV-VAR due to its superior model fit as demonstrated in Section 5.1.

[Insert Figure 1 about here]

### 3.1.3. Revision in Analysts' Forecasts (AF)

In addition to the VAR-type approach, which is widely used in the literature, we also consider an alternative approach that based on analysts' forecasts in constructing the news

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<sup>14</sup> The online appendix of CV (2004) show that the cash flow and discount rate betas in their modern sample period are affected by the small sample bias given that the state variables of the VAR system are highly persistent. The estimated risk premium associated with the cash flow beta reduces greatly and reverse the conclusion that the cash flow beta earns a higher premium than the discount rate beta.

series. Using the standard return decomposition of Campbell and Shiller (1988a) and Campbell (1991), Khimich (2017) define the  $N_{CF}$  as the revision in the analysts' forecasts of the  $ROE$  ( $FROE$ ) instead of discounted sum of clean-surplus  $ROE$ <sup>15</sup> as proposed in Cohen et al. (2002) and Vuolteenaho (2002), and back out the  $N_{DR}$  as residual, as shown below:

$$r_{t+1} - E_t(r_{t+1}) = (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j ROE_{t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \rho^j r_{t+1j} \quad (9)$$

$$r_{t+1} - E_t(r_{t+1}) = \sum_{j=1}^{\infty} \rho^j (FROE_{t+1,t+1+j} - FROE_{t,t+1+j}) - \Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1j}$$

$$N_{CF,t+1} = \sum_{j=1}^{\infty} \rho^j (FROE_{t+1,t+1+j} - FROE_{t,t+1+j}) \quad (10)$$

$$N_{DR,t+1} = -\Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1j} = -[r_{t+1} - E_t(r_{t+1}) - N_{CF,t+1}] \quad (11)$$

where  $\Delta E_{t+1} \sum_{j=1}^{\infty} \rho^j r_{t+1j}$  represents the variations in the discounted sum of expected returns and

is computed as the difference between unexpected stock market returns and  $N_{CF}$ . The  $N_{CF}$  is defined as the discounted sum of the revision in analysts' forecasts, which is the difference between the  $ROE$  forecasts generated at time  $t$  ( $FROE_{t,t+1+j}$ ) and the  $ROE$  forecasts produced at time  $t + 1$  ( $FROE_{t+1,t+1+j}$ ). Similar to the previous approaches, the discounting factor used in this method is assumed to be  $0.95^{1/12}$  as well. The  $ROE$  forecasts are computed as  $FROE_{t+i} = FEPS_{t+i} / BV_{t+i-1}$ . Despite cash flow news in equation (10) requiring an infinite sum of  $FROE$ , forecasts of only up to twelve years are used for practical purpose following the work of Gebhardt et al. (2001) and Khimich (2017). The mean of one- and two-year-ahead  $EPS$  forecast,  $FEPS$ , is readily available from the Bloomberg. The three-year-ahead  $FEPS$  can be computed as  $FEPS_{t+3} = FEPS_{t+2} (1 + LTG)$ , where  $LTG$  represents the long-term  $EPS$  growth rate predicted by analysts. In line with other literature, the  $FROE$  beyond three years is assumed to revert to the median of aggregate  $ROE$ . As for the book value,  $BV$ , it can be forecasted based on clean surplus principle as  $BV_{t+i} = BV_{t+i-1} + FEPS_{t+i} - FDPS_{t+i}$ , where

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<sup>15</sup> Clean surplus accounting requires that the variation in the book value is calculated by subtracting the net dividends from earnings in order to ensure that gains and losses affecting the earnings are accounted in the computation.

$$FDPS_{t+i} = FEPS_{t+i} \times k = FEPS_{t+i} \times (D_{t+i-1} / E_{t+i-1}) \quad (12)$$

$k$  is the current dividend payout ratio computed as a ratio of dividend over earnings. This study accounts for the possibility that the accounting information is publicly available only after forecasts have been made by taking the lagged term of dividend and earnings in the construction of  $k$ .

This measure reflects the markets' expectation about the future cash flow and hence analysts maybe optimistic in their forecasts. Zhu and Niu (2016) indeed find that investor sentiment does affect the predicted earnings growth rate. Also, Hribar and McInnis (2012) reveal that one-year-ahead  $FEPS$  and  $LTG$  tend to be more optimistic during high sentiment periods. Meanwhile, Easton and Monahan (2005) claim that the low-quality of analysts' forecasts is the culprit for the lack of reliability of the accounting-based measures as a proxy to the expected returns. They found that accounting-based proxies are less reliable in estimating the expected stock returns when the  $LTG$  is high. On the other hand, all proxies are positively correlated to the expected returns when the  $LTG$  is low and *ex-post* analysts' forecasts have lower errors. Their findings are hence a manifestation that analysts' forecasts could be rational at some times but irrational at another times. Since the analysts' forecasts are of low quality and the degree of rationality of analysts' forecasts is varying over time, the analysts' forecasts may not be a good proxy to the cash flow expectations.

### 3.2 Four-beta model

Given the estimated  $N_{CF}$  and  $N_{DR}$ , the rational and irrational components of the market news can be retrieved from the following regressions<sup>16</sup>:

$$N_{CF,t} = \alpha + \sum_{i=0}^{12} \beta_i S_{t-i}^{TV} + \varepsilon_t \quad (13)$$

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<sup>16</sup> Since investor sentiment is highly persistent, the lagged terms of investor sentiment index have been incorporated in the regressions to avoid the omitted variable bias. Although we could have selected the lagged terms based on the information criterion, such as AIC or BIC, but the information criterion tends to select the parsimonious model of up to one lagged term. This may not truly reflects the effect of investor sentiment on the unexpected stock market returns given the persistence feature of investor sentiment. Our main goal here is to capture as much as possible the sentiment effect in the irrational new series, and to clean out as much sentiment effect as possible from the rational new series. Hence, investor sentiments from the past twelve months are incorporated in equations (14) and (15). Besides that, capturing the previous twelve months' sentiment could remove (or reduce) any possible seasonal effect of investor sentiment on stock returns.



$$N_{DR,t} = \varpi + \sum_{i=0}^{12} \gamma_i S_{t-i}^{TV} + \eta_t \quad (14)$$

where  $N_{CF,t}$  and  $N_{DR,t}$  are cash flow and discount rate news, respectively, estimated based on the framework of return decomposition.  $S^{TV}$  denotes the time-varying weighted version of Baker and Wurgler (2006) investor sentiment index<sup>17</sup>. The residual series,  $\varepsilon_t$  and  $\eta_t$ , represent the rational component of the cash flow news,  $N_{CF,t}^R$ , and the discount rate news,  $N_{DR,t}^R$ . The irrational component of the cash flow news,  $N_{CF,t}^{IR}$ , and the discount rate news,  $N_{DR,t}^{IR}$ , are simply the fitted value of the above regressions. A four-beta model is then constructed to measure the sensitivity of stock returns to each of these news series, where each beta is defined as follow:

$$\beta_{i,CF}^{IR} = \frac{Cov(r_{i,t}, N_{CF,t}^{IR})}{Var(r_{M,t}^e)} \quad (15)$$

$$\beta_{i,CF}^R = \frac{Cov(r_{i,t}, N_{CF,t}^R)}{Var(r_{M,t}^e)} \quad (16)$$

$$\beta_{i,DR}^{IR} = \frac{Cov(r_{i,t}, N_{DR,t}^{IR})}{Var(r_{M,t}^e)} \quad (17)$$

$$\beta_{i,DR}^R = \frac{Cov(r_{i,t}, N_{DR,t}^R)}{Var(r_{M,t}^e)} \quad (18)$$

CPV (2010) compute the cash flow beta and discount rate beta based on the scaled news series in order to adjust the regression coefficients of different scales to a common scale - variance of excess market return,  $Var(r_M^e)$ , so that  $\beta_{i,CF}$  and  $\beta_{i,DR}$  sum up to the market beta.

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<sup>17</sup> Extending the framework of Baker and Wurgler (2006), we construct an enhanced investor sentiment index that allows the loading assigned to each sentiment component in the index to vary over time.

Adapting their approach, each news series is first scaled by the ratio of the variance of excess market return,  $Var(r_M^e)$ , to the variance of each news series as shown below:

$$SN_{CF,t}^{IR} = N_{CF,t}^{IR} \times \frac{Var(r_{M,t}^e)}{Var(N_{CF,t}^{IR})} \quad (19)$$

$$SN_{CF,t}^R = N_{CF,t}^R \times \frac{Var(r_{M,t}^e)}{Var(N_{CF,t}^R)} \quad (20)$$

$$SN_{DR,t}^{IR} = N_{DR,t}^{IR} \times \frac{Var(r_{M,t}^e)}{Var(N_{DR,t}^{IR})} \quad (21)$$

$$SN_{DR,t}^R = N_{DR,t}^R \times \frac{Var(r_{M,t}^e)}{Var(N_{DR,t}^R)} \quad (22)$$

where  $SN_{CF,t}^{IR}$  and  $SN_{CF,t}^R$  are irrational and rational scaled cash flow news series;  $SN_{DR,t}^{IR}$  and  $SN_{DR,t}^R$  are irrational and rational scaled discount rate news series. Then, we perform the following regression to empirically estimate the four betas:

$$r_{i,t} = \alpha + \beta SN_{j,t}^E + \varepsilon_t, \quad SN_{j,t}^E = \{SN_{CF,t}^{IR}, SN_{CF,t}^R, SN_{DR,t}^{IR}, SN_{DR,t}^R\} \quad (23)$$

where  $r_{i,t}$  represents the portfolio's log returns and  $SN_{j,t}^E$  denotes one of the four scaled news series computed from equations (19) – (22). The  $\beta$  is the corresponding beta estimates for each news series depending on which scaled news series is used to perform the above regression.

The following equations show that cash flow beta,  $\beta_{i,CF}$ , comprises of the irrational cash flow beta,  $\beta_{i,CF}^{IR}$ , and the rational cash flow beta,  $\beta_{i,CF}^R$ ; whereas the discount rate beta,  $\beta_{i,DR}$ , constitutes of irrational discount rate beta,  $\beta_{i,DR}^{IR}$ , and the rational discount rate beta,  $\beta_{i,DR}^R$ .

$$\beta_{i,CF} = \beta_{i,CF}^{IR} + \beta_{i,CF}^R \quad (24)$$

$$\beta_{i,DR} = \beta_{i,DR}^{IR} + \beta_{i,DR}^R \quad (25)$$

The summation of the cash flow and discount rate betas adds up to the market beta (see CPV, 2010).

### 3.3 Pricing of the four-beta model

If a market is not fully dominated by the long-term risk averse investors but both risk averse and risk seeking investors constitute the market players instead, the rational and irrational risks could carry different premiums. Distinguishing between the sensitivity of stock returns to the rational and irrational components in each channel allows us to answer the question: does stock market rewards investors for bearing both types of risks in each channel? We perform the FMB regression in order to estimate the risk premium associated with each beta risk in our four-beta model. Concretely, we run the following cross-sectional regression at each month  $t$ .

$$R_{i,t}^e = \lambda_{CF,t}^{IR} \cdot \beta_{i,CF}^{IR} + \lambda_{CF,t}^R \cdot \beta_{i,CF}^R + \lambda_{DR,t}^{IR} \cdot \beta_{i,DR}^{IR} + \lambda_{DR,t}^R \cdot \beta_{i,DR}^R + e_{i,t} \quad (26)$$

where  $R_{i,t}^e$ , denotes the simple excess returns on portfolio  $i$  at time  $t$ ,  $\beta_{i,CF}^{IR}$  is the irrational cash flow beta on portfolio  $i$ ,  $\beta_{i,CF}^R$  is the rational cash flow beta on portfolio  $i$ ,  $\beta_{i,DR}^{IR}$  is the irrational discount rate beta on portfolio  $i$ ,  $\beta_{i,DR}^R$  is the rational discount rate beta on portfolio  $i$ .  $\lambda_{j,t}$  and  $e_{i,t}$  are the cross-sectional slope coefficients and the pricing errors, respectively, at each time  $t$ . The risk premium associated with each beta factor is the time-series average of the cross-sectional slope coefficients, *i.e.*  $\hat{\lambda} = \frac{1}{T} \sum_{t=1}^T \hat{\lambda}_{j,t}$ . We then test whether this estimated risk premium is significantly different from zero with the use of Newey-West standard errors in order to account for the autocorrelated  $\hat{\lambda}_i$ . We compare the performance of our four-beta model, constructed based on different approaches, to the CAPM and the CV's two-beta models, where all models are estimated based on the FMB procedure. A better asset pricing model will deliver a higher adjusted cross-sectional  $R^2$  and a relatively lower average pricing errors.

As mentioned by Lewellen, Nagel and Shanken (2010), the freely estimated risk premia will inflate the cross-sectional explanatory power, in terms of the cross-sectional  $R^2$  statistic. Therefore, following Campbell et al. (2018) and Ho and Hung (2009), we impose theoretical restriction on the asset pricing specification. Particularly, we restrict the zero-beta rate to be equal to the risk-free rate and the risk premium equals the excess returns of the factor.

## 4. Data and descriptive statistics

### 4.1 VAR (and TV-VAR) data

To ensure our result is comparable to CV (2004), four state variables, which are excess market returns ( $r_M^e$ ), the term yield spread ( $TY$ ), the price-earnings ratio ( $PE$ ) and the small-stock value spread ( $VS$ ), employed in their study are used in our VAR model to decompose the excess market returns into  $N_{CF}$  and  $N_{DR}$  for the period 1969:12 – 2014:12<sup>18</sup>. These four state variables are also employed in the literature (see Celiker, Kayacetin, Kumar and Sonaer, 2016; Chen and Zhao, 2009; CPV, 2010; Campbell, Giglio and Polk, 2013, CGP hereafter). The detailed construction of each variable is discussed as follow.

The  $r_M^e$  is computed as the monthly log stock market return minus the log risk-free rate. The stock market return is the value-weighted S&P 500 index returns (inclusive of dividends) retrieved from the Center for Research in Security Press (CRSP). The risk-free rate is 3-month Treasury-bill rate. The second variable,  $TY$ , is constructed using the series different from the work of CV (2004). As in Welch and Goyal (2008), we compute the  $TY$  as the difference between the yield on U.S. long-term government bond and the yield on U.S. Treasury-bills, expressed in percentage point. This measure is included in the VAR framework since it captures the business cycle (Fama and French, 1989), where the  $TY$  is low (high) at the peaks (troughs) of the business cycle. Since expected stock market return is countercyclical, low term spread is hence predicts low expected returns during the expansion

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<sup>18</sup> The news series are computed from December 1969 in order to account for the effect of investor sentiment which has its first data point in December 1968, from the previous twelve months on the news series. Hence, we exclude the sample from December 1968 to November 1969 in the VAR estimation in order to exclude the possibility that the unexpected returns during this period is affected by investor sentiment from the past twelve months, of which is not available prior to December 1968.

period and vice versa. The risk-free rate together with both series used in the *TY* computation are retrieved from Professor Amit Goyal website<sup>19</sup>.

Next, the *PE* ratio is defined as the log-smoothed *PE* ratio. Specifically, it is constructed as ratio of the price of the S&P 500 index to a ten-year moving average of the earnings of S&P 500 index. This ratio is log transformed. The S&P 500 index and market's earnings series are retrieved from the website of Professor Robert J. Shiller<sup>20</sup>. To avoid any look-ahead bias in our data construction, we smooth the latest ten years of earnings series. *PE* ratio is included since it reflects the expected future returns; a high *PE* ratio indicates a low expected returns. The same series of *PE* ratio is used in the direct proxy approach as well.

The last state variable, *VS*, is computed using the book-to-market ratio and monthly return series of stock portfolios obtained from the website of Professor Kenneth French<sup>21</sup>. As stated on his website, the portfolios, which are constructed at the end of each June, are the intersections of two portfolios formed on size (market equity, *ME*) and three portfolios formed on the ratio of book equity to market equity (*BE/ME*). The size breakpoint for year *t* is the median NYSE market equity at the end of June of year *t*. *BE/ME* for June of year *t* is the book equity for the last fiscal year end in *t* – 1 divided by *ME* for June of *t* – 1. We employ the value-weighted average of *BE/ME* computed for June of year *t* to the June of year

*t*+1<sup>22</sup>. The value-weighted average of *BE/ME* is calculated as  $\sum_{i=1}^{12} [ME_i \times (BE / ME)] / \sum_{i=1}^{12} ME_i$ ,

where *i* represents a month from June of *t* to June of *t* + 1. The *BE/ME* breakpoints are the 30<sup>th</sup> and 70<sup>th</sup> NYSE percentiles. The monthly small-stock value spread is simply the difference in *BE/ME* between the small-value and small-growth stocks.

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<sup>19</sup> <http://www.hec.unil.ch/agoyal/>

<sup>20</sup> <http://www.econ.yale.edu/~shiller/data.htm>

<sup>21</sup> [http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html)

<sup>22</sup> We consider the *BE/ME* constructed from the *ME* for June of *t* – 1 instead of the *ME* for December of *t* – 1, such as that of the test asset portfolios in Section 4.3. This is because the *VS* constructed from portfolios sorted based on the *BE/ME* computed from the *ME* for June of *t* – 1 is more correlated to that of in the CV (2004) data file (*i.e.* 0.935 for the *VS* computed based on the *BE/ME* computed from the *ME* for June of *t* – 1 vs. 0.898 for the *VS* computed based on the *BE/ME* computed from the *ME* for December of *t* – 1).

## 4.2 Analysts' forecasts data

The calculation of the analyst forecast of returns on equity (*FROE*) requires the forecast of earnings per share (*FEPS*) and the book value (*BV*). The data and the construction of the numerator of *FROE*, which is the *FEPS*, is first discussed. The mean of one- and two-year-ahead *FEPS* can be obtained from the Bloomberg Estimates (BEst)<sup>23</sup>. In line with the literature, the forecast fiscal period value associated with a fiscal year is adopted in this study. These forecasts are available from January 1990. The two-year-ahead *FEPS* from January to March of 2005 are missing<sup>24</sup>. These missing values are filled by using linear interpolation.

Although three-year-ahead *FEPS* does provided by BEst, many missing values (about 22% of the series) have been found in the series. Hence, the three-year-ahead forecast is computed based on the available one- and two-year-ahead forecasts and long-term earnings per share (*EPS*) growth rate (*LTG*) following Gebhardt et al. (2001) and Khimich (2017). *LTG* from BEst is the estimated Compounded Annual Growth Rate (CAGR) of the operating *EPS* over the company's next full business cycle, which is typically three to five years. The *LTG* series from BEst is only available from July 2005, the missing values prior to this month is filled by computing the composite growth rate underlying in the one- and two-year-ahead *FEPS* as in Gebhardt et al. (2001). The *FEPS* beyond year 3 is interpolated linearly up to year 12, of which the *FEPS* is the median of *ROE* computed as the moving median of past *ROEs*<sup>25</sup>.

The denominator of *FROE* – *BV* – is computed using the *FEPS*, forecasted dividend per share (*FDPS*) and historical *BV*, which is used to construct the one-year-ahead *BV*. The historical dividends and earnings used to obtain the *FDPS* as well as the historical *BV* are retrieved from the Bloomberg terminal in order to ensure forecast value is congruent to the realized value. For the historical dividend, this study opts for the most recently announced

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<sup>23</sup> Although most studies employed the analysts' earnings forecasts retrieved from the Institute of Broker Estimates System (IBES), we retrieve those forecasts from BEst as we do not have the access to the IBES.

<sup>24</sup> The missing values are due to the lower coverage factor, where less than 50% of the securities have their *FEPS* reported from brokers for these few months. Hence, BEst is unable to aggregate the forecasts of the underlying constituent stocks to the index level forecast.

<sup>25</sup> This procedure follows closely to that of Gebhardt et al. (2001) and Khimich (2017), whose study focuses on the firm level. Instead of using the median industry *ROE*, this study uses the median value of aggregate *ROE*. A window of five years is employed to estimate the median of *ROE*.

gross dividend in order to truly reflect the dividends received by investors. Meanwhile, the basic EPS is employed in this study.

There is a caveat using the analysts' earnings forecasts from BEst. As described in the footnotes 23, the aggregate value of forecast is provided by BEst as long as more than 50% of the securities have their *FEPS* reported by brokers. Nevertheless, we are unsure about the actual percentage of securities that have the brokers' estimates in each month, *i.e.* the actual coverage percentage could vary in between 51% to 100%. Apart from the issue of the coverage factor at the index level, the coverage at the individual securities also have the same issue, where the minimum number of brokers' estimates required for each security is one. Hence, the consistency of the earnings forecasts for the S&P500 index across months could be a question. Besides that, the earnings forecasts available on the BEst has a much shorter sample period as compared to the earnings forecasts provided by IBES, which can be traced back to the year 1983. Therefore, these issues could affect the results of the four-beta model computed from the analysts' forecasts approach and the results obtained here may not fully comparable to previous studies which employed the forecasts from IBES.

#### *4.3 Test asset portfolios*

With the market news series computed from different approaches, the four-beta model can be tested on the 25 portfolios formed based on firm size and book-to-market ratio. These portfolios are downloaded from the Professor Kenneth French's website. The portfolios are the intersection of five portfolios sorted based on *ME* and five portfolios sorted based on *BE/ME* ratio, constructed at the end of each June. *BE/ME* for Jun of year  $t$  is the book equity for the last fiscal year end in  $t - 1$  divided by *ME* for December of  $t - 1$ . The breakpoints for size and *BE/ME* are the NYSE quintiles. To perform the regression (27), we compute the monthly simple excess returns on our test asset portfolios.

#### *4.4 Descriptive statistics of data*

Table 1 reports the descriptive statistics for the data used to decompose the excess market return into different news series based on VAR and analysts' forecasts approach, in

panel A and B, respectively. The correlations among the state variables of VAR model are presented at the bottom of panel A.

[Insert Table 1 around here]

Overall, the descriptive statistics of VAR's state variables are in line with that of reported in CV (2004). The  $r_M^e$  has a mean of 0.4% and a median of 0.8%. The standard deviation of  $r_M^e$  is 4.5%. These statistics of  $r_M^e$  are in line with the literature (see CV, 2004; Huang et al., 2015; Neely, Rapach, Tu and Zhou, 2014). Among all the state variables, *PE* ratio has the highest mean value, whereas *TY* varies the most around its mean according to the standard deviation measure. The first-order autocorrelation measure indicates that all state variables but  $r_M^e$  are highly persistent with autocorrelation statistics of greater than 0.9.

The correlations among VAR state variables, as shown in the bottom of panel A, are highly significant even though the magnitude of each correlation is relatively low. The highest correlation of 0.269 is reported for the relationship between *VS* and *PE*. Although the sign of the correlation between  $r_M^e$  and *PE* is inconsistent with the correlation reported in CV (2004), it is consistent with CGP (2013), who include a relatively latest sample period as compared to CV (2004). The stock market returns is positively associated with *PE* ratio since the current high (low) price inflates (deflates) the contemporaneous stock return.

Panel B shows that the average of historical ROE over the sample period of 1990:01 – 2014:12 is 0.131, a value lower than the average forecasts of ROE across different forecast horizons, which range from 0.136 at twelve-year-ahead forecast to 0.167 at one-year-ahead forecast. This reflects that analysts generally produce optimistic forecasts, which is consistent with the literature (*e.g.* Chen, Da and Zhao, 2013; Hribar and McInnis, 2012). Nevertheless, the median forecast of ROE is not higher than the median historical ROE beyond six-year-ahead forecasts, reflecting pessimistic forecasts to a certain extent. Also, the difference between the mean historical ROE and the mean forecasts of ROE (*i.e.* approximation of forecast errors) decreases with the forecast horizon. This could probably indicate that the forecasts of ROE are not entirely optimistic across different forecast horizons, instead the initial optimistic bias could be offset by the pessimistic bias (or a reduction in the optimistic bias) in the long-horizons forecast. Therefore, a neutral tone (*i.e.* no bias) in the news series



could be obtained when we add up the revision in the analysts' forecasts over time to form the news series as shown in equation (10).

## 5. Empirical results

### 5.1 The estimation of the VAR model

Table 2 presents the average parameter estimates of the first-order TV-VAR model retrieved via OLS estimation across 470 windows. The values in the square brackets underneath the parameter estimates are their heteroscedasticity and autocorrelation consistent (HAC) standard errors. Each regression regresses a state variable on five independent variables, which are a constant and the lagged terms of four state variables, in each window. The table also reports the mean adjusted  $R$ -squared obtained from the OLS estimation across different windows in the last column.

[Insert Table 2 around here]

As shown in the table, the coefficient sign of each state variable in the first row is consistent with the literature. First, the term yield spread, although statistically insignificant, predicts positively the excess market returns, consistent with Campbell and Thomson (2008), Fama and French (1989), Keim and Stambaugh (1986), Rapach, Ringgenberg and Zhou (2016). Second, both price-earnings ratio and value spread predict negatively and significantly the excess market returns, with the coefficient of  $-0.1067$  and  $-0.040$ , respectively<sup>26</sup>. Finally, the excess market return has a negative but statistically insignificant coefficient of  $-0.033$ , displaying a moderate price reversal, which is consistent with Campbell, Giglio and Polk (2013)<sup>27</sup>.

The regressions of other state variables depict that most state variables are highly autocorrelated with their coefficients greater than 0.90, except that of  $VS$ . Their

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<sup>26</sup> Campbell and Thompson (2008), Neely et al. (2014), Rapach et al. (2016) find that earnings-price ratio predict aggregate stock returns positively. The negative relation between value spread and future excess market returns can be interpreted as lower future stock market returns is a result of the overvaluation of current small-growth stocks, which creates a larger value spread. Brennan, Wang and Xia (2002) report this negative relationship between  $VS$  and future excess market returns.

<sup>27</sup> The full sample estimation, however, shows that stock market returns exhibit a momentum with the lagged excess market returns has an insignificant coefficient of 0.053, in line with CV (2004).

autocorrelation coefficients are highly significant at 1% level. These results are consistent with the autocorrelation statistics reported in the Table 1. For the *TY*, we notice that other state variables also significantly predict the future *TY* at 1% significance level. In contrast to CV (2004), we find that not only the excess market return, but *VS* also predicts significantly the next month's *PE* ratio. The coefficient of excess market returns, 0.435, is highly significant at 1% level; the coefficient of *VS*, -0.027, is statistically significant at 10% level. The negative association between *VS* and future *PE* is similar to that of *VS* and excess market return. Increase in the value spread denotes that the small growth stocks are currently overvalued, which forecast a lower expected stock market return and *PE* ratio. As for the value spread, it is also highly predictable by the lagged one month of other state variables, which are excess market returns and *PE* ratio, apart from its own lagged term.

Since *TY*, *PE* and *VS* are highly persistent, the regression model for these state variables have higher explanatory power, in terms of the mean adjusted  $R^2$  statistics, as compared to the return regression. One important thing to note is that a higher adjusted  $R^2$  (7.90%) is obtained when the expected returns is estimated on a rolling window basis. Contrarily, the constant VAR approach produces an adjusted  $R^2$  of 1.62%, close to 2% as reported in CV (2004) and Maio (2013a, 2013b) who estimate the VAR over the full sample period as well. This indicates that allowing the coefficients to pick up the dynamics of the state variables over time improves the predictive power of the return regression.

Table 3 reports the attributes of the two components of unexpected returns –  $N_{CF}$  and  $N_{DR}$ . Both news series are the values of the news series average across different estimation windows at each time  $t$ . The top panel of the table presents the variance-covariance matrix of  $N_{CF}$  and  $N_{DR}$ , and the values in bracket are the standard deviation and correlation of the new series. It shows that the variance of  $N_{DR}$  is slightly higher than that of  $N_{CF}$ , which are 0.52% and 0.43%, respectively. This finding suggests that  $N_{DR}$  has a slightly more important role in the stock market returns, in line with most literature (*e.g.* Botshekan et al., 2012; Campbell, 1991; CV, 2004; CPV, 2010; Campbell et al., 2018). Furthermore, the discount rate news and cash flow news have a correlation of 0.8268, This indicates that a good (bad) cash flow news is associated with an increase (a drop) in the discount rate could be attributable to the mispricing, where investors extrapolate the favourable (unfavourable) stock prices movement resulted from good (bad) cash flow news in forming their expected return as discussed in

Barberis, Greenwood, Jin and Shleifer (2015), or be due to the risk-based explanations as discussed in Cohen et al. (2002, p. 442).

[Insert Table 3 around here]

The bottom panel depicts the time-series average of the linear function coefficients that connect the VAR shocks to the news series.  $e1'+e1'\lambda$  is the cash flow news function and  $e1'\lambda$  is the discount rate news function. Given the linear function coefficients of  $N_{CF}$  and  $N_{DR}$ , only the innovation in  $r_M^e$  will be mapped differently into both news series. The additional term of  $\mathbf{e1}'\mathbf{u}_{t+1}$  in the  $N_{CF,t+1}$ , where  $\mathbf{e1}'$  has a unity value for only the first element in the vector, adds the value of  $r_M^e$  shocks (zero) to the  $N_{CF,t+1}$  when the innovation of  $r_M^e$  (other state variables) is mapped into the  $N_{CF,t+1}$ . As such, shocks in *TMS*, *PE* and *VS* have the same contributions to both news series.

The coefficients of the linear functions capture the long-run effect of the shock in each state variable to the  $N_{CF}$  and  $N_{DR}$ . Therefore, the shocks of a state variable have a greater contribution to the discount rate news when that variable's coefficient is higher in the return predictive regression (CPV, 2010). Consistent with the coefficients shown in the first row Table 2, *TMS*, which has the least impact on the expected excess market returns, also contributes the least (in absolute value) to both news series. On the other hand, shocks in *PE* receive the greatest weight (in absolute value) in the computation of both news series. The innovation of  $r_M^e$  have a positive long-run effect on the  $N_{CF}$  (0.6188), but a negative long-run effect on the  $N_{DR}$  (0.3812). This suggests that an increase in the  $r_M^e$  shocks leads to an increase in the CF expectations and a decrease in the DR expectation. Whilst the shocks in *TMS* and *PE* carry a negative effect on the news series, *VS* contribute positively to the news series<sup>28</sup>.

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<sup>28</sup> The coefficients of the *TMS* and the *VS* shocks produced by constant VAR have a positive and negative effect, respectively, to the new series, consistent with CV (2004), CPV (2010), and Campbell et al. (2018). Therefore, the time-series average of the coefficients of these two shocks having an opposite sign under the TV-VAR framework could be due to the outliers in a few windows.

## 5.2 The four news terms

Table 4 presents the correlations among the four scaled news series, which are irrational cash flow news ( $SN_{CF}^{IR}$ ), rational cash flow news ( $SN_{CF}^R$ ), irrational discount rate news ( $SN_{DR}^{IR}$ ), and rational discount rate news ( $SN_{DR}^R$ ). We notice that irrational news series and rational news series are uncorrelated, suggesting that our model is able to disentangle the irrational news series from the rational news series. Besides that, in line with Table 3, the cash flow and discount rate are positively correlated in both rational and irrational channels. The irrational news series have a correlation of 0.896; whereas the rational news series have a correlation of 0.825.

[Insert Table 4 around here]

Figure 2 plots the four smoothed news series (only for Figure 2) estimated based on the equations (15) to (18) with the  $N_{CF}$  and  $N_{DR}$  retrieved from the TV-VAR specification. Each row corresponds to one news series, where the first row presents the irrational cash flow news series ( $SN_{CF}^{IR}$ ), the second row depicts the rational cash flow news series ( $SN_{CF}^R$ ) followed by the irrational discount rate news series ( $SN_{DR}^{IR}$ ), and the rational discount rate news series ( $SN_{DR}^R$ ) is in the last row. The shaded bars denotes the NBER-dated recessions.

The illustration supports the correlation reported in the Table 4, where the rational and irrational news series in both cash flow and discount rate channels generally do not seem to have any relationship. Meanwhile, in each rational and irrational channel, the cash flow and discount rate move in the opposite directions for most of the periods, pushing the stock market prices in the same direction. The variation in the  $N_{CF}$  seems to be mainly picked up by the variation in  $SN_{CF}^{IR}$  since the  $SN_{CF}^R$  wavering around the zero value (*i.e.* no apparent shocks), except a few periods where the  $SN_{CF}^R$  has noticeable variation. Unlike the cash flow channel, both irrational and rational discount rate news vary considerably over time, justifying the greater role of discount rate news (*i.e.* greater variance) in the stock market as shown in Table 3.

In the early 1970s, both irrational cash flow and irrational discount rate news exhibit greater fluctuations during the oil shock, especially the huge increase of  $SN_{DR}^{IR}$  moving from the negative new to the positive news, which reflects the deterioration of investor sentiment that penalize the expected returns heavily. During this period, the  $SN_{CF}^R$  drops from positive values to negative values, indicating the expectations towards the future fundamental cash flow is revised downward.

The recession in the early 1980s can be explained by the declining rational and irrational cash flow given that both  $SN_{CF}^{IR}$  and  $SN_{CF}^R$  experience a sharp decline during this period. Also, there is an increase in both rationally and irrationally expected discount rate. Together, all forces push down the stock market price during this period. The expansion state after this period can be described by the negative rational discount rate news and an improvement in the irrational expectations of future cash flow. Turning to the recession in 1991, CV (2004) claim that it is a profitability recession caused by unfavourable move in the expectations about the future cash flow. As shown in Figure 1, the bad cash flow news in 1991 is mainly ensued from the declining irrational expectations of future cash flow since changes in the rationally expected cash flow are near zero.

The technology boom in the late 1990s can be justified not only by the decreases in both irrational and rational discount rate, but also an increase in the irrational expectations of the future cash flow, in line with Ofek and Richardson (2002) findings. The decrease in both rational and irrational discount rates also shows that the lower discount rate during this period is not merely due to the improving sentiment as claimed by Campbell, Giglio and Polk (2013), but the risk-based explanation of the discount rate also plays a role here. As for the  $SN_{CF}^R$ , the revision in the rational cash flow expectation remains positive even though the magnitude of the news is reducing. Similar causes but in the opposite direction are accountable for the burst of the dot-com bubble, where investors increased both discount rates and the high irrational expectations of future cash flow is now reversed. Prior to the recession in late 2000s, investors irrationally expected a high future payoff, which can be seen from the positive  $SN_{CF}^{IR}$ . Later, the recession in 2007 – 2009 could be ascribed to the negative  $SN_{CF}^R$ , supported by the declining irrational expectations of cash flow as well as the

positive  $SN_{DR}^{IR}$ . Overall, the four news terms align with the fluctuation in the US stock market.

### 5.3 The four-beta model

This section examines the sensitivity of portfolio returns to changes of both irrational and rational expectations in CF and DR channels. If CPV assumptions are correct, the null hypothesis that the irrational cash flow betas and rational discount rate betas are not significantly different from zero, *i.e.*  $H_0 : \beta_{CF}^{IR} = 0$  and  $H_0 : \beta_{DR}^R = 0$ , should not be rejected.

#### 5.3.1. TV-VAR approach

The baseline results of TV-VAR model are reported in Table 5. Each panel in Table 5 corresponds to the four betas, which are irrational cash flow beta ( $\beta_{i,CF}^{IR}$ ), rational cash flow beta ( $\beta_{i,CF}^R$ ), irrational discount rate beta ( $\beta_{i,DR}^{IR}$ ), and rational discount rate beta ( $\beta_{i,DR}^R$ ), computed based on the scaled news series computed from equations (15) to (18) for 25 portfolios sorted according to firm size ( $ME$ ) and book-to-market ( $BE/ME$ ) ratio. The summation of  $\beta_{i,CF}^{IR}$  and  $\beta_{i,CF}^R$  equals to the  $\beta_{i,CF}$ ; whereas  $\beta_{i,DR}^{IR}$  and  $\beta_{i,DR}^R$  add up to the  $\beta_{i,DR}$ . The betas are the slope coefficients obtained via OLS regression for the period of 1969:12 – 2014:12 and the Newey-West  $t$ -statistics (automatic bandwidth selection) are reported underneath the beta estimates in the square brackets.

[Insert Table 5 around here]

The panel A of Table 5 shows that the  $\beta_{i,CF}^{IR}$  of all portfolios but one have a value of greater than 0.010. Furthermore, about half of the portfolios considered here are significantly affected by the changes in the irrationally expected cash flow at a significance level of at least 5%. Looking at the magnitude of  $\beta_{i,CF}^{IR}$ , the results show that growth stocks respond stronger to the fluctuations in the irrational cash flow expectations, as do small stocks as compared to large stocks. On the other hand, variations in rational expectations of future cash flows, in general, do not significantly affect the stock price movements, as shown in panel B, even though the rational cash flow beta estimates (in absolute term) are higher than the irrational

cash flow beta estimates for most portfolios, except for the small stocks. As for the discount rate channel, panel D depicts that all assets considered in this study react significantly (1% significance level) to the rational discount rate news but are not significantly affected by the shocks in the irrational discount rate as shown in panel C. While none of the  $\beta_{i,DR}^{IR}$  has an estimate of greater than 0.10, the  $\beta_{i,DR}^R$  estimates range from the value of 0.588 to 1.104, which is more than five times of the  $\beta_{i,DR}^{IR}$ .

To provide comparison on the relative importance of  $\beta_{i,CF}^{IR}$  and  $\beta_{i,DR}^{IR}$ , we compute the proportion (in absolute term) of the irrational news to the rational news in each cash flow and discount rate channel, as shown in Table 6. Panel A report the results for the proportion of irrational beta relative to the rational beta in the cash flow channel (*i.e.*  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$ ); panel B shows the proportion of irrational beta relative to the rational beta in the discount rate channel (*i.e.*  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$ ). Again, if the assumptions from previous literature that cash flow news links to fundamental and discount rate news is sentiment driven is true, we would expect that  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$  is higher than  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$ . However, the results as shown in Table 6 reveal the opposite findings. The proportion of the irrational cash flow beta relative to the rational cash flow beta (*i.e.*  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  in panel A) is higher than the proportion of irrational discount rate beta to the rational discount rate beta (*i.e.*  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$  in panel B) across all portfolios. For instance, the small-growth stocks have a  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  of 0.607 but a  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$  of only 0.003. This shows that the irrational component constitutes a higher proportion in the cash flow risk than in the discount rate risk. Indeed, a considerable source of cash flow risk is originated from the irrational cash flow beta when  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  exceeds 0.50. Therefore, the findings from this comparison suggest that the cash flow news and hence the cash flow risk are not solely links to fundamental.

As mentioned in the introduction, although the expected returns could be more accurately characterized by the TV-VAR approach, this approach does introduce small sample bias that could possibly affect our conclusion. In view of this, we present the results retrieved from the constant VAR approach in the following section. Our conclusion could be

strengthen if both approaches with the trade-off between “correctness” and bias produce commonality in the beta estimates.

### 5.3.2. Constant VAR approach

The sensitivity of portfolio returns to the changes in the four news series retrieved from the constant VAR approach is presented in Table 7. The four betas are arranged in the similar order as in Table 5. It is apparent from panel A that all portfolio returns, regardless of the firm size and the *BE/ME* ratio, are sensitive to the variations in the irrationally expected cash flow. The value of  $\beta_{i,CF}^{IR}$  ranges from 0.012 to 0.023. Despite the responsiveness of stock prices to the irrational cash flow news is not of great amount, irrational cash flow beta estimates are significant at 5% level in 22 out of 25 portfolios, and the beta estimates of the other three portfolios are significant at 10% level. Thus, this result again shows that the variations in the irrational expectation of cash flow should not be ignored and  $H_0 : \beta_{CF}^{IR} = 0$  is rejected. Meanwhile, panel B shows that  $\beta_{i,CF}^R$  of all portfolios sorted based on size and *BE/ME* ratio are highly significant at 1% level, with the magnitude of beta estimates higher than 0.40 across the board.

Turning to the discount rate betas, panel D shows that discount rate betas in the rational channel are highly significant at 1% level, consistent with the results shown in Table 5. Similar to  $\beta_{i,CF}^R$ ,  $\beta_{i,DR}^R$  of each portfolio is greater than 0.40. As for the irrational discount rate betas in the panel C, all portfolios returns are significantly affected by the fluctuations in the irrationally expected discount rates with the *t*-statistics of  $\beta_{i,DR}^{IR}$  are greater than 1.96 in half of the portfolios considered. This finding shows that stocks’ exposure to the systematic variation in discount rate news is attributable to both rational and behavioural explanations and hence  $H_0 : \beta_{DR}^R = 0$  is rejected.

Comparing  $\beta_{i,CF}^{IR}$  against  $\beta_{i,DR}^{IR}$  as shown in Table 7, we find that the irrational cash flow beta estimates are greater than the irrational discount rate beta across 25 portfolios. Besides that, the null hypothesis that stock price is not sensitive to the variations in the irrationally expected cash flow can be rejected at a more stringent significance threshold as



compared to the irrational discount rate beta. To provide further comparison, the ratio of irrational cash flow beta to the rational cash flow beta (*i.e.*  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  in panel A) and the ratio of irrational discount rate beta to the rational discount rate beta (*i.e.*  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$ ) in panel B), as in the previous section, are shown in Table 8. The findings are in line with that of in the Table 6. The irrational beta has a higher proportion in the cash flow channel than in the discount rate channel. Even though the magnitude of  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  has seen a drop across the board, this does not affect the conclusion that a relatively greater irrational component is embedded in the cash flow risk than in the discount rate risk.

### 5.3.3. TV-VAR vs. VAR

Comparing the beta estimates constructed from both approaches, we can confirm that the assets' returns are not immune to the changes in the irrationally expected cash flow and the rational discount rate since  $\beta_{i,CF}^{IR}$  and  $\beta_{i,DR}^R$  are the two beta estimates that remain significant under both approaches; whereas  $\beta_{i,CF}^R$  and  $\beta_{i,DR}^{IR}$  lose their significance under the TV-VAR approach. Hence, our findings do not support the claim that cash flow news is fundamentally driven and discount rate news merely reflect the changes in sentiment<sup>29</sup>. CV (2004) find that the cash flow and discount rate beta estimates are biased downward when there is a small sample bias. Looking at both Table 5 and 7, we notice that the downward bias in the beta estimates could be due to  $\beta_{i,CF}^R$  and  $\beta_{i,DR}^{IR}$ , where both estimates experience a severe downward bias when the news series are retrieved from the TV-VAR as shown in Table 5.

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<sup>29</sup> As a robustness check, the TV-VAR is estimated on a longer rolling window size – 180 months. The results are robust to the change in the rolling window size of the TV-VAR framework.  $\beta_{CF}^{IR} / \beta_{CF}^R$  is greater than  $\beta_{DR}^{IR} / \beta_{DR}^R$ , confirming that cash flow channel is an important medium through which the sentiment affect the stock prices. Furthermore, the number of portfolios that are sensitive to the irrational cash flow and rational discount rate betas has seen an increase. Furthermore, the degree of significance of the rational discount rate beta increases across the board. In general, the results tend towards the findings obtained from the constant VAR model.

#### 5.3.4. AF approach

The beta estimates produced under the analysts' forecast approach is reported in Table 9. As shown in panel B and D, the stock returns are sensitive to the movement in the rational expectations, regardless of the cash flow or discount rate channel. However, panel C shows that only a few portfolios are affected by the change in the irrational discount rate. The result of insignificant irrational cash flow beta across 25 portfolios as in panel A is inconsistent with our intuition that the variation in irrationally expected cash flow computed from AF approach will significantly affect the stock prices, since previous studies found that analysts' earnings forecasts contain systematic error (La Porta, 1996) and could be overoptimistic (Abarbanell and Bernard, 1992; De Bondt and Thaler, 1990; Hribar and McInnis, 2012). This could be due that the sample period used for this approach is limited, where the data starts from January 1990, and the analysts' forecast in this study is sourced from the Bloomberg Earnings Estimates (BEst), which has a few issues as discussed in the Section 4.2, instead of the Institutional Brokers' Estimate System (IBES), which is commonly used to retrieved the analysts' earnings forecasts in past studies. Nevertheless, the findings from three approaches agree that the discount rate new is not merely affected by sentiment, rather risk-based explanations seems to play a more important role.

[Insert Table 9 around here]

#### 5.3.5. Summary

In general, our findings suggest that the assumptions of cash flow news is driven by fundamentals and discount rate news is driven by sentiment are less appropriate seeing that the asset prices consistently move in response to the changes in  $SN_{CF}^{IR}$  and  $SN_{DR}^R$  according to TV-VAR and VAR approaches, and that  $\beta_{i,CF}^{IR}$  is a relatively more important systematic risk component as compared to  $\beta_{i,DR}^{IR}$ . Apart from validating the assumptions, our results do provide support to the findings of previous literature that stock prices are affected by the irrational expectations of the future cash flow (*e.g.* Engelberg et al., 2018; Kim, Ryu and Seo, 2014; LSV, 1994; Park, 2005), and the rationally expected future returns (*e.g.* Bansal et al., 2012; Campbell and Cochrane, 1999; Lettau and Ludvigson, 2001; Gabaix, 2012).

Previous studies claimed that the cash flow of value stocks are fundamentally riskier than growth stocks since value stocks consistently have poor earnings (Fama and French, 1993; 1995), and hence their higher expected returns is a compensation for the high fundamental cash flow risk (see Campbell, Polk and Vuolteenaho, 2010; Campbell and Vuolteenaho, 2004). Yet, our results based on TV-VAR and VAR approaches show that the values stocks are not fundamentally riskier considering that the rational cash flow beta, which reflects the fundamental cash flow risk, of the value stocks is lower than that of the growth stocks across different size quintiles. Hence our results do not support the risk-based explanation to a certain extent. Rather, it is consistent with LSV (1994) viewpoint that investors extrapolate the past growth rates, leading them to overreact to the news and to misprice the stocks.

#### *5.4 The prices of four betas*

The results from previous section show that stocks are sensitive not only to the rational movement in the news series, but also to the irrational fluctuation in the news series. Therefore, the standard asset pricing test is employed to investigate how the four betas are being priced cross-sectionally by performing the Fama-Macbeth regression (FMB) as shown in equation (26). We compare the performance of our four-beta models against the Capital Asset Pricing Model (CAPM) and two-beta model in the pricing of risks. The CV's two-beta model used in this sub-section is constructed based on the the TV-VAR approach. Meanwhile, the regression estimates allows us to examine the relative importance of the premium investors allocate to each component of our four-beta model.

Figure 3 provides a visual examination on the model fit of different asset pricing models. The figure plots the average realized returns in excess of risk-free rate (vertical axis) against the average fitted excess returns (horizontal axis). The average fitted excess returns is the fitted value of equation (27) estimated for the period of December 1969 to December 2014, except the four-beta model (AF) where the sample period covers from January 1990 to December 2014. The dots in each graph are the 25 portfolios sorted based on  $ME$  and  $ME/BE$ , represented by the two-digit number labelled next to each data point. The first digit denotes the size quintiles ( $ME$ ) and the second digit represents the book-to-market quintiles

$(BE/ME)^{30}$ . If a model explains 100% of the variation in the cross-section of average stock returns, all data points would lie exactly on the 45-degree line.

[Insert Figure 3 around here]

The CAPM and CV's two-beta models are plotted on the top of the Figure 3, and the other three graphs are the four-beta models constructed under different approaches, which are the time-varying VAR (TV-VAR), the Vector Autoregressive Model (VAR) and the analysts' forecast (AF). Of all the five models, CAPM apparently performs the worst as the model seems to predict the average excess return of different portfolios far too away from the 45-degree reference line. Although the model fit is improved in the two-beta model, we notice that the data points produced by our four-beta model generally have much smaller spread from the reference line, regardless of the approach used to compute the four-beta model. A noticeable exception can be seen from the small-growth portfolio (labelled as 11), where it has the greatest distance from the 45-degree line not only in our models, but also in the CAPM and two-beta models.

To confirm our visual evidence, the cross-sectional regression result of each model is presented in Table 10. The risk premia estimates ( $\lambda$ ) of all asset pricing models but four-beta model (AF) model are computed from December 1969 to December 2014. The sample period covered for the four-beta model (AF) spans from January 1990 to December 2014 due to the data availability. The result of each asset pricing model is presented in column (1) to (5). Each row is presented with the point estimates of a particular risk component (*i.e.* risk premium) associated with its heteroscedasticity and autocorrelation consistent t-statistics (shown in the square bracket). The tests statistics of adjusted- $R^2$  and pricing errors, measured as root-mean-squared-pricing-errors (*RMSPE*), the mean-pricing-errors (*MPE* (%)), are used to evaluate the performance of each asset pricing model as presented in the last three rows.

[Insert Table 10 around here]

The first column shows that the explanatory power of CAPM on the cross-sectional stock returns is only 0.3% although the model produces a positive risk price for the market

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<sup>30</sup> For instance, the double-digit 11 denotes the small-growth portfolio, *i.e.* ME1BM1 portfolio.

beta (*i.e.* 7.2% per annum for  $\lambda_M$ <sup>31</sup>) that is highly significant at 1% level. This result suggests that the CAPM model is unable to price the cross-sectional stock returns and produces largest average pricing errors (*e.g.* 0.024 *RMSPE*), consistent with the visual representation as shown in Figure 3. When the market beta is disentangled into cash flow and discount rate beta, the adjusted- $R^2$  statistic even though does improved tremendously to 19.4%, as shown in the column (2), the root-mean-squared-pricing-errors improved by less than 0.005 to 0.021. Besides that, only the discount rate beta carries a positive risk premium (*i.e.* 10.8% per annum for  $\lambda_{DR}^{IR}$ ) that is significant at 1% level. Although CV (2004) and Garrett and Priestley (2012) report a higher cross-sectional  $R^2$  statistics (*i.e.* more than 40%) for the two-beta model, Botshekan et al. (2012) who included a more recent sample period present a much lower cross-sectional  $R^2$  statistic, which is less than 10%<sup>32</sup>.

Columns (3) to (5) show the results of pricing tests for the four-beta models constructed using news series retrieved from TV-VAR, VAR, AF approach, respectively. Our four-beta models perform better than the CAPM and CV's two-beta models in terms of the cross-sectional adjusted- $R^2$  statistic. The higher adjusted- $R^2$  statistics together with much lower average pricing errors suggest that the four-beta models are able to explain the variation in the average stock returns at the cross-sectional level as compared to the other two asset pricing models, reassuring the visual evidences shown in the Figure 3. This improvement is a result of decomposing the cash flow and discount rate betas into irrational and rational components that yields a richer description of the risk components faced by different stocks.

Of the three four-beta models, the TV-VAR approach performs the best with the highest cross-sectional adjusted- $R^2$  (*i.e.* 37.8%), and all risk components but the rational cash flow risk ( $\beta_{CF}^R$ ) are priced at 1% significance level. Also, it produces the least *RMSPE* of 0.016. It is noteworthy that the irrational cash flow risk,  $\beta_{CF}^{IR}$ , is consistently and significantly priced in the cross-section of stock returns under the VAR frameworks. It is also worth noting

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<sup>31</sup> The point estimates reported in the Table 10 are monthly risk premium estimates. To obtain the annual risk premium in the percentage term, simply multiply the point estimates by 1200.

<sup>32</sup> The modern sample period of CV (2004) is from July 1963 to December 2001 and Garrett and Priestley (2012) employ the annual data from 1928 to 2001; whereas Botshekan et al (2012) extend the sample period of CV (2004) to December 2008.

that the irrational betas are important components in describing the cross-sectional stock returns given that the irrational cash flow and irrational discount rate betas consistently carry a larger risk premium (in the absolute terms) as compared to their rational counterparts across all four-beta models, *i.e.*  $\lambda_{CF}^{IR} > \lambda_{CF}^R$  and  $\lambda_{DR}^{IR} > \lambda_{DR}^R$ . Given the relative importance of irrational risk premia, and since the expected returns is the product of the beta estimates and the corresponding risk prices, investors should pay attention to the assets that are more sensitive to the variation in the irrational news, especially those with high irrational cash flow beta, despite the magnitude of the irrational betas is smaller than that of the rational betas.

In accord with our expectation, irrational cash flow and irrational discount rate risks (*i.e.*  $\beta_{CF}^{IR}$  and  $\beta_{DR}^{IR}$ ) consistently carry a negative risk premium across three different approaches used in retrieving the news terms. There are a few potential rational and behavioural explanations to the negative risk premium as discussed below.

The pricing of irrational beta risks could be well reflected by the pricing of lottery-like stocks' characteristics. Investors who trade on sentiment tend to invest in lottery-like stocks, and hence, the returns of lottery-like stocks tend to be driven by investor sentiment (Carpentier, Romon and Suret, 2018; Fong, 2013; Fong and Toh, 2014). At the same time, Kumar (2009) define the lottery-like stocks as stocks typically with high idiosyncratic volatility (IVOL) and positive idiosyncratic skewness (SKEW)<sup>33</sup>. As IVOL is highly correlated to the market volatility, Barinov (2018) shows that IVOL carries a negative risk premium, an insurance investors pay to shield from an unfavourable move in the market volatility – consistent with the rational explanation. Hence, lottery-like stocks with high IVOL beta and tends to earn lower expected returns since it hedges against the market volatility risk.

As for the positive SKEW, Barberis and Huang (2008) claim that prospect utility investors overweight the small probability of the huge gains of lottery-like stocks<sup>34</sup>, and

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<sup>33</sup> Fong (2013) also mention that other characteristics of lottery stocks are similar to that of the sentiment-driven stocks, such as small, young, unprofitable, distressed and high growth stocks.

<sup>34</sup> Apart from overweighting the tail probability, investors face limited downside risk with the asymmetric payoff structure of the lottery stocks. Pessimistic investors will stand on the side line when investment opportunities deteriorates and the stock prices will not be punished severely, but they may be greatly rewarded when optimistic investors actively purchase the lottery-like stocks.

hence investors are willing to pay a price for the lottery-like stocks, hoping for a potentially huge payoff, and accept a lower average excess returns – consistent with the behavioural explanation<sup>35</sup>. Therefore, the positive SKEW is priced negatively in the cross-section of expected returns (Bali, Cakici, and Whitelaw, 2011; Boyer, Mitton and Vorkink, 2010; Lin and Liu, 2018). Given that lottery-like stocks are affected by investor sentiment, and that the characteristics of the lottery-like stocks are negatively priced, the negative risk premia of the sentiment-induced irrational betas could be a manifestation of the negative risk premia associated with those characteristics – IVOL and SKEW.

As a whole, stocks with high irrational betas would command a negative risk premium. Since growth stocks consistently have lower average returns after controlling for the size effect<sup>36</sup>, the pricing of our model on the irrational beta risks is consistent with the mean return differences and beta estimates. The lower average returns of growth stocks can be seen to correspond to the higher irrational cash flow beta as compared to value stocks in Table 5. Meanwhile, Table 7 shows that the beta estimates constructed under the VAR approach have both the irrational cash flow and discount rate betas of growth stocks higher than that of the value stocks. Therefore, growth stocks that are more sensitive to the change in the irrational expectations earn lower average excess returns.

### 5.5 Robustness Checks

*Different window size.* – The TV-VAR is re-estimated on a longer rolling window size – 180 months – in order to see if the four beta estimates and the associated conclusion regarding the CPV’s assumptions is affected. The results are robust to the change in the rolling window size of the TV-VAR framework.  $\beta_{CF}^{IR} / \beta_{CF}^R$  is greater than  $\beta_{DR}^{IR} / \beta_{DR}^R$ , confirming that cash flow channel is an important medium through which the sentiment affect the stock prices. The number of portfolios that are sensitive to the irrational cash flow and rational discount rate betas has seen an increase. Furthermore, the degree of significance

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<sup>35</sup> In fact, investors prefer any securities that exhibit positive skewness in return distribution, such as, premium bond (Lobe and Hölzl, 2008; Pfiffelmann; 2013) and lottery-linked deposit account (Guillen and Tschoegl, 2002).

<sup>36</sup> The return differences for value-growth portfolios and large-small portfolios are not reported in this paper, but is available upon request.

of the rational discount rate beta increases across the board. In general, the results tend towards the findings obtained from the constant VAR model.

*Sub-sample analysis.* – Recognizing the fact that the beta is not constant across time, we also conduct a sub-sample analysis to investigate if the beta estimates and their associated risk premia produced under the TV-VAR approach change across different sub-sample periods. Single break test of Andrew (1993) is employed in order to locate the single break point on the cash flow and discount rate news<sup>37</sup>. To avoid having different break points for each portfolio, we treat the 25 portfolio as the representative of the stock market and the break point identified on the stock market return-beta relation is applied to all test asset portfolios considered in this study.

In general, the findings on the beta estimates in this sub-sample analysis again do not support the claim made by CPV (2010). The effect of the variation in the rational discount rates on the stock prices is robust across different sub-sample periods. Besides that, some stocks are sensitive to the variation in the irrational cash flow expectations as well in the second sub-sample period. Thereby, our findings suggest that  $Cov(r_{i,t}, SN_{CF,t}^{IR}) \neq 0$  and  $Cov(r_{i,t}, SN_{DR,t}^R) \neq 0$ . As for the risk prices of each risk factor,  $\beta_{i,CF}^R$  and  $\beta_{i,DR}^R$  have lost their influence in the cross-sectional asset pricing since the risk premia estimates of 10 basis points per month for  $\beta_{i,CF}^R$  and 40 basis points per month for  $\beta_{i,DR}^R$  are statistically insignificant. Contrarily, irrational beta risks remain significantly priced in the cross-section of average stock returns. Both irrational betas have almost similar risk premia (20% to 30% per month in the absolute term). Our findings suggest that the significant risk premia for irrational beta risks are robust across different sub-sample periods. Moreover, our model consistently outperforms its counterparts in pricing the cross-sectional of stock returns. This observation is consistent with the results reported in the full sample period. Therefore, the structural break in the beta estimates does not affect much on the pricing of risk for the four-beta model and its performance superior than the CAPM and two-beta model even though the explanatory power has dropped in the second sub-sample period.

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<sup>37</sup> The break date identified for the cash flow and discount rate beta are September 1997 and February 1998, respectively. In view of the need of having the consistent break point for both betas, we exclude the sample period from September 1997 to January 1998. Therefore, the first sub-sample period covers from December 1969 to August 1997, and the second sub-sample period spans from February 1998 to December 2014.



*Adding extra test asset portfolio.* - To address the concern of the potential inflated risk premium and the cross-sectional  $R^2$  in the four-beta model, this section follows the suggestion of Lewellen et al. (2010) by including 10 momentum sorted portfolios (10MOM) or 10 industry sorted portfolios (IND) apart from the 25 size-*BE/ME* sorted portfolios as a robustness check. Overall, the baseline results for the asset pricing of the four-beta model are robust to the inclusion of additional test asset portfolios. The four-beta model (constructed under the TV-VAR framework) is not only able to price the 25 size and *BE/ME* sorted portfolios, but also able to describe the average stock returns of momentum portfolios, which are known to be anomalous to other models, and of industry-sorted portfolios better than the CAPM and the two-beta model. Besides that, the irrational (rational) risk factors are negatively (positively) and significantly priced in the four-beta model when either 10MOM or 10IND are included, except the rational cash flow risk which is not significantly priced when 10IND are included as test asset portfolios.

*Control for Fama-French factors.* - To arrive at a firm conclusion regarding the pricing performance of the four-beta model, a set of Fama-French factors is incorporated as control variables in the FMB regression. Specifically, we consider the Fama and French (1993) three factors (FF-3), Carhart (1997) four factors (FFC-4)<sup>38</sup>, and Fama and French (2015) five factors (FF-5) as control variables in separate regressions. The pricing of the four betas generally stands firm even after controlling for the Fama-French factors. Meanwhile, the findings also demonstrate the superiority of the four-beta model in explaining the differences in the stock returns at the cross-sectional level given that the four-beta model consistently delivers the highest adjusted cross-sectional  $R^2$  statistic as compared to its counterparts (e.g. the adjusted- $R^2$  statistics of CAPM, two-beta and four-beta models after controlling for FF-5 are 46.7%, 47.3% and 49.3%, respectively).

Therefore, the baseline results that (1) the four-beta model explains the cross-sectional variation of asset returns better than the CAPM and the two-beta model, (2) the irrational beta risks consistently priced at the cross-sectional level and carry negative premia, and (3) the

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<sup>38</sup> Since Carhart (1997) four-factor model is built on the Fama and French (1993) three-factor model, this study terms it as Fama-French-Carhart four factors (FFC-4).

rational beta risks demand a positive premium, are robust to the inclusion of different control variables.

## 6. Empirical application: Anomalies test

Given the usefulness of the four-beta model in explaining the cross-sectional variation of the average returns, it would be interesting to know whether the model can explain various equity anomalies documented in the literature. We consider a set of equity anomalies employed in Campbell et al. (2018).

Following Campbell et al. (2018), this study measures the ability of an asset pricing model in explaining the anomalies by comparing the abnormal returns,  $\alpha$ , produced by different models. A model is claimed to have a superior ability in explaining the anomalies whenever it produces lower  $\alpha$ . The abnormal returns of an anomaly portfolio is calculated as  $\alpha_i = \bar{R}_i^e - E(R_i^e)$ , where  $\bar{R}_i^e$  is the sample mean excess return and  $E(R_i^e)$  is the predicted excess returns, following Campbell et al. (2018).

The out-of-sample evaluation of anomalies is considered here. The risk premium estimate associated with each beta in the CAPM, the CV's two-beta and the four-beta models are not re-estimated. Instead, the risk premium estimates are derived from 35 test asset portfolios, which are 25 size-*BE/ME* sorted portfolio (FF25) plus 10MOM<sup>39</sup>, and denote the risk premium corresponds to a particular beta as  $\lambda_{35,\beta}$ . The betas of each asset pricing model are recomputed to measure the sensitivity of anomaly portfolio returns to the (1) market returns in the CAPM model, (2) cash flow and discount rate news in the two-beta model, (3) rational and irrational news series of each cash flow and discount rate channel in the four-beta model. Each of these betas is denoted as  $\hat{\beta}_{i,k}$ , where  $i$  represents one of the anomaly portfolios and  $k$  corresponds to one of the news series (or market returns for the CAPM model). The predicted excess returns of an anomaly portfolio is then computed as

$$E(R_i^e) = \sum \hat{\lambda}_{35,\beta} \times \hat{\beta}_{i,k}.$$


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<sup>39</sup> The risk premia estimates computed based on FF25+10MOM are employed because this set of test asset portfolios addresses the issue of the strong factor structure of FF25 and delivers higher adjusted cross-sectional  $R^2$  statistics for the two-beta and four-beta models constructed under TV-VAR framework. In fact, the general conclusion obtained in this section is unaffected by the use of FF25 or FF25+10IND as the test asset portfolios.

The pricing performance of each model on the anomaly portfolios are shown in Table. The mean excess returns ( $\mu$ ) and the standard deviation ( $\sigma$ ) of anomaly portfolios are reported in the second and third column, respectively, followed by the four-beta estimates. The last three columns present the abnormal returns of the anomalies (expressed in percentage term) computed based on CAPM ( $\alpha_{CAPM}$ ), two-beta model ( $\alpha_{2B}$ ), and four-beta model ( $\alpha_{4B}$ ). The second column shows that all anomaly strategies have a positive excess returns, which can be partially explained by the negative loadings associated with the irrational risk components. As reported in Section 5.4 and 5.5, the irrational betas are robustly priced across assets and consistently command a negative risk premium. Therefore, assets that are highly sensitive to the irrational risk components should earn a lower returns and vice versa. The positive excess returns of anomaly portfolios, except the RMRF, is hence justifiable on the ground of their negative irrational beta in the cash flow and/ or discount rate channel.

[Insert Table 11 around here]

The abnormal returns produced by CAPM,  $\alpha_{CAPM}$ , are positive across all anomalies but RMRF, which has the abnormal returns of slightly below zero. This implies that the realized returns are generally greater than the expected returns as predicted by CAPM. As mentioned earlier, a model performs better than the other model in explaining a particular anomaly when the estimated alpha has reduced. The results show that CV's two-beta model does not perform any better than the CAPM as the abnormal returns (in the absolute term) of about half of the anomalies are higher with the two-beta model. Contrarily, the four-beta model is seen to perform better than the CAPM and the two-beta model, where the model produces the lowest abnormal returns for more than half of the anomaly portfolios. The four-beta model performs exceptionally well for the anomaly strategies of idiosyncratic volatility (VAR and RESVAR), HML, and BETA with more than 80% reduction in  $\alpha$  relative to the other two models is observed. Exceptions where the four-beta model does not perform as well as the other two models in explaining the anomalies include the returns on RMRF, SMB, RMW, LTR and ACC.

To get a clearer picture, the anomalies test results are summarized in Table 12Table. The mean absolute alphas,  $\bar{\alpha}$ , generated by different models across all anomaly strategies, the portfolios of Fama and French (1993) three-factor model, the portfolios of Fama-French-Carhart (1997) four-factor model, and the portfolios of Fama and French (2015) five-factor

model are presented. Both the raw and scaled mean absolute alphas are shown in the table. The scaled alpha is estimated by rescaling the mean absolute alpha of each anomaly to have the same variability as RMRF.

The last column clearly depicts that the four-beta model has the lowest mean absolute alpha, both scaled and unscaled, across all anomaly strategies. The anomaly returns left unattended by the four-beta model are about 0.30% and 0.10% for unscaled and scaled alpha, respectively. The two-beta model, on the other hand, have near zero reduction in the mean absolute alpha relative to the CAPM across all strategies. In fact, the unscaled mean absolute alpha of the two-beta model is slightly higher than that of the CAPM, which is 0.55% versus 0.53%. As for the Fama and French (1993) three-factor anomalies, the mean absolute abnormal returns of the four-beta model decrease from the CAPM's 0.21% to 0.11%. Similar results are observed when the Fama-French-Carhart (1997) four-factor anomalies and the Fama and French (2015) five-factor anomalies are considered, where the four-beta model has about 0.10% reduction in the unscaled alpha as compared to CAPM.

As emphasized in Lewellen et al. (2010), a model can be viewed as successful even if it explains only one or two anomalies as long as the factor structure issue is addressed by expanding the test asset portfolios. As such, the four-beta model not only can be viewed as a success, but also outperforms the other two models in describing the average returns of anomalies given the great shrink in the anomaly returns averaged across all strategies. This result implies that the factors in the four-beta model capture well the risk exposures that describe the average stock returns.

## **7. Conclusion**

This study decomposes the cash flow and discount rate betas of the CV's two-beta model into a four-beta model by taking into consideration of the effect of irrational expectations on the stock prices. Thereby, the four-beta model comprises of four components, which are the rational and irrational components in each cash flow and discount rate beta. By using the four-beta model, we evaluate the assumptions applied in previous studies that, especially the claims made by CPV (2010), the cash flow news is fundamentally driven and the discount rate news is mainly driven by investor sentiment. If their claims are true, two

null hypotheses should not be rejected: (1) covariances between stock returns and the shocks in the irrational cash flow expectations (*i.e.* irrational cash flow beta) is zero, and (2) covariances between stock returns and the shocks in rational discount rate expectations (*i.e.* rational discount rate beta) is zero. Besides that, we assess whether each of the four components in the four-beta model is priced in the cross-sectional regression.

Our baseline results are based on the cash flow and discount rate news series generated from the time-varying VAR (TV-VAR) approach on account of the fact that the VAR parameter estimates are unstable. We support the baseline results with the findings obtained from the constant VAR approach. Empirically, this study reveals that the covariances of the asset returns with the irrational cash flow news and rational discount rate news are indeed significantly different from zero. Thus, the null hypotheses as stated above are rejected with confidence. In fact, only the irrational cash flow beta and the rational discount rate beta consistently have positive and significant estimates under both TV-VAR and constant VAR frameworks. Meanwhile, the structural break analysis reveals that the response of asset prices to the variation in the rational discount rate expectations is robust across different sub-sample periods. Also, the significant effect of the irrational cash flow news on the stock prices is observed in the latest sub-sample period. All these findings reinforce the conclusion that the assumptions made in previous studies may not that appropriate.

The asset pricing test of the four-beta model against CAPM and the CV's two-beta models shows that our model greatly improves the explanatory power, in terms of the cross-sectional  $R^2$  and the pricing errors, of the other two asset pricing models. The cross-sectional regression also shows that irrational beta risks (*i.e.* irrational cash flow and irrational discount rate betas) as well as the rational discount rate beta are priced in the cross-section of stock returns. The sub-sample analysis also find that these three components are consistently priced across different sub-sample periods even though the explanatory power of the four-beta model has been affected slightly in the second sub-sample period. Whilst the irrational beta risks command negative premia, the rational discount rate beta risk carries a positive premium across different stocks. These findings are robust to the inclusion of additional test asset portfolios as well as to the control of a set of Fama-French factors. Further empirical

evaluation of the four-beta model shows that the model is useful in explaining a set of anomalies.

Overall, our findings imply that the variation in the cash flow expectation is not merely link to the fundamental factors, likewise, the variation in the discount rate news which should not be treated as mispricing news at all times. Therefore, the cash flow beta is not fundamentally driven; the discount rate beta is not sentiment driven. Furthermore, given a better model fit in the cross-section of stock returns is achieved by our four-beta model, the asset pricing model in the future should incorporate both irrational and rational elements into one model instead of studying their implication on the pricing of risk separately. The pricing of the four betas also suggests that investors are willing to pay a price for the stocks that are sensitive to the irrational risk factors but require a risk premium for bearing the rational risk factors.

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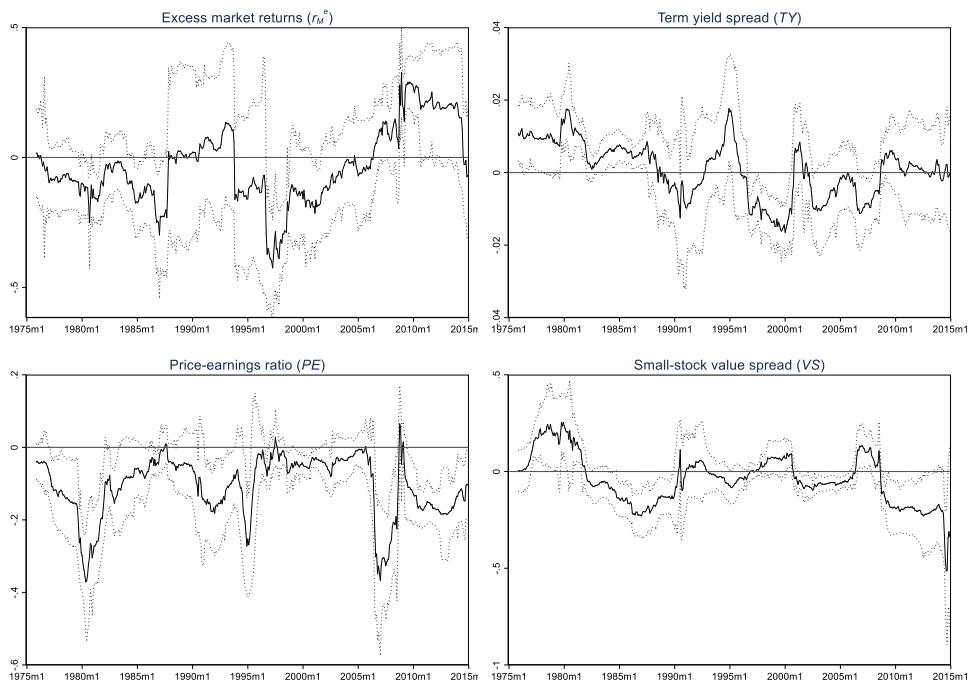
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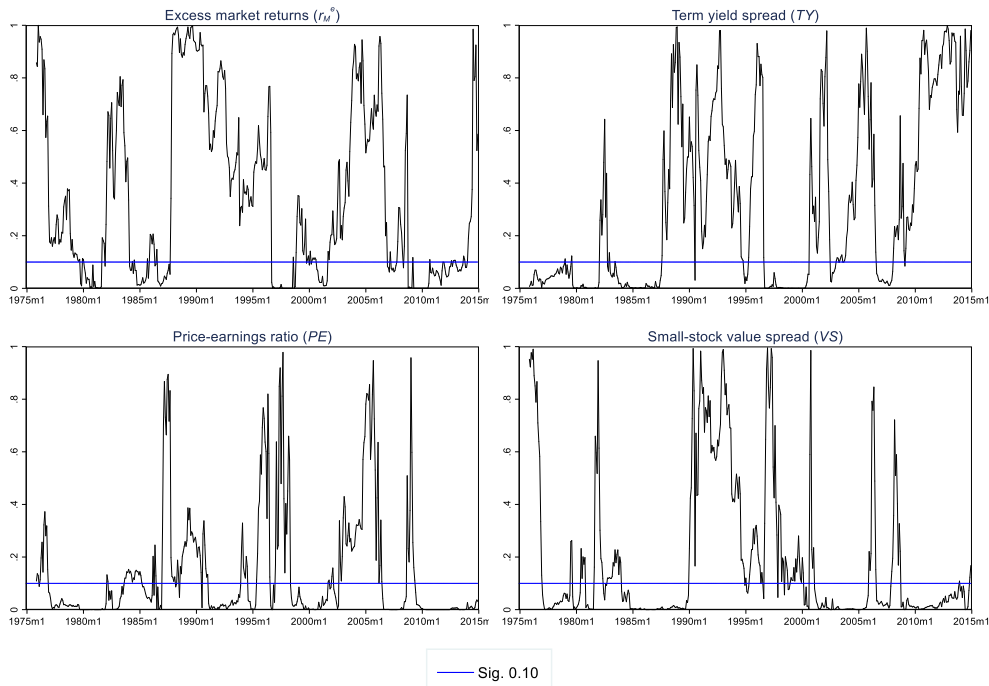
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## Panel A: Rolling Coefficient Estimates

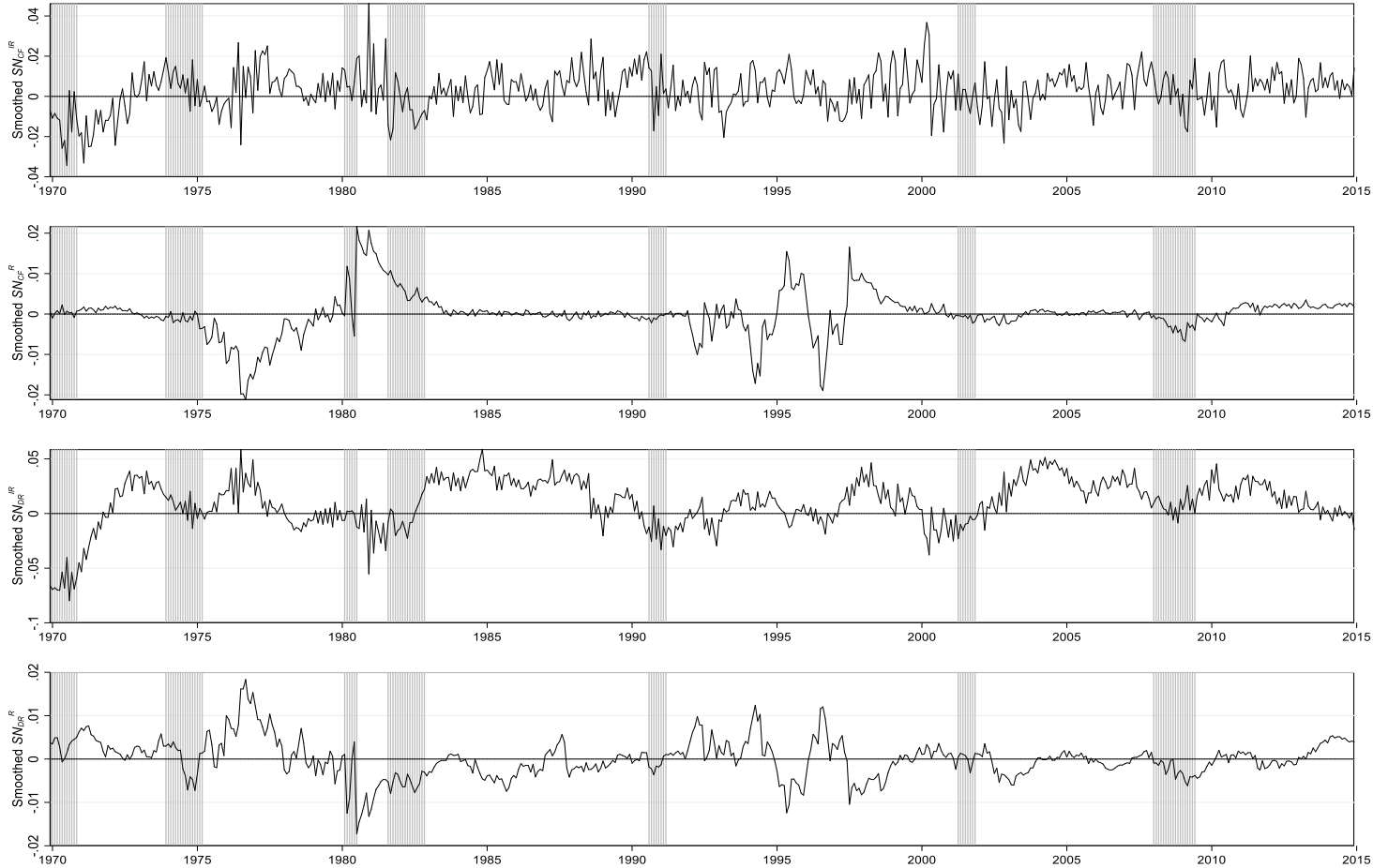


## Panel B: Rolling $p$ -values



**Figure 1**  
**72-month rolling estimates for return regression**

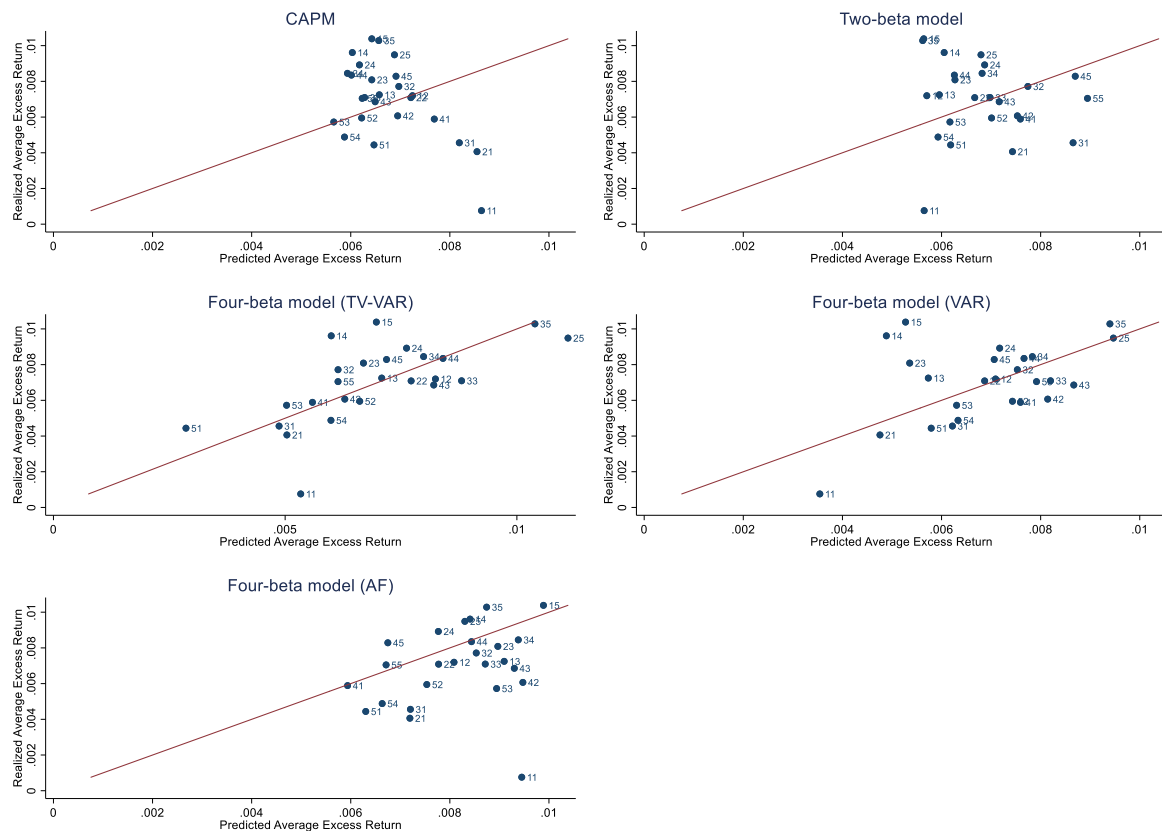
This figure plots the rolling regression estimates for the return regression on a rolling window basis estimated from December 1969 to December 2014. The state variables used to predict the excess market return are the lagged terms of the excess market return ( $r_M^e$ ), the term yield spread ( $TY$ ), the price-earnings ratio ( $PE$ ) and the small-stock value spread ( $VS$ ). Panel A depicts the rolling slope coefficient of each state variable associated with its 95% confidence interval represented by dotted lines. Panel B plots the rolling  $p$ -value for the estimated coefficient of each state variable. The horizontal line in panel B denotes the significance level of 10%.



**Figure 2**

**Four scaled news series of the four-beta model**

This figure depicts the four scaled news series estimated from equations (15) to (18) based on the TV-VAR specification for the sample period of 1969:12 – 2014:12. These news series are irrational cash flow news ( $SN_{CF}^{IR}$ ), rational cash flow news ( $SN_{CF}^R$ ), irrational discount rate news ( $SN_{DR}^{IR}$ ), and rational discount rate news ( $SN_{DR}^R$ ), presented in each row of the figure. These news terms are smoothed under the specification of an exponentially weighted moving average:  $MA_t(SN_j^E) = 0.08SN_{j,t}^E + (1-0.08)MA_{t-1}(SN_j^E)$ , where  $SN_j^E$  is the respective news series. The shaded bars represent the recession period as dated by NBER.



**Figure 3**  
**Realized vs. fitted average excess returns**

This figure plots the realized average excess return against the fitted (or predicted) average excess returns on 25 portfolios sorted based on firm size ( $ME$ ) and book-to-market ( $ME/BE$ ) ratio (represented as dots in the figure) for different asset pricing models: Capital Asset Pricing Model (CAPM), CV's two-beta model, four-beta models computed with the news series retrieved from the time-varying VAR (TV-VAR), the constant VAR and the analysts' forecast (AF) approach. The predicted average excess returns are estimated from equation (27) for the period of 1969:12 – 2014:12, except the four-beta model (AF), which has the sample period spans from 1990:01 – 2014:12. The number labelled next to each data point represents the portfolios sorted according to  $ME$  and  $BE/ME$  ratio (e.g. double-digit 15 denotes small-value portfolio).

**Table 1****Summary statistics of data**

<i>Panel A: VAR Approach</i>						
	<i>Mean</i>	<i>Median</i>	<i>SD</i>	<i>Min</i>	<i>Max</i>	$\rho(1)$
$r_M^e$	0.004	0.008	0.045	-0.248	0.149	0.057
<i>TY</i>	2.059	2.230	1.493	-3.650	4.550	0.946
<i>PE</i>	3.070	3.092	0.367	2.298	3.891	0.994
<i>VS</i>	1.495	1.487	0.145	1.231	1.952	0.945
<i>Correlations</i>	$r_M^e$	<i>TY</i>	<i>PE</i>	<i>VS</i>		
$r_M^e$	1.000					
<i>TY</i>	0.086**	1.000				
<i>PE</i>	0.031	0.083*	1.000			
<i>VS</i>	-0.106**	0.255***	0.269***	1.000		
<i>Panel B: Analysts' Forecasts Approach</i>						
	<i>Mean</i>	<i>Median</i>	<i>SD</i>	<i>Min</i>	<i>Max</i>	
<i>DPS</i>	1.672	1.509	0.831	0.538	4.680	
<i>EPS</i>	47.963	43.590	28.532	4.660	108.710	
<i>BV</i>	375.824	334.500	172.111	165.570	739.050	
<i>ROE</i>	0.131	0.144	0.046	0.027	0.193	
<i>LTG</i>	0.133	0.133	0.056	-0.133	0.464	
<i>FROE1</i>	0.167	0.168	0.019	0.117	0.204	
<i>FROE2</i>	0.165	0.165	0.013	0.136	0.194	
<i>FROE3</i>	0.152	0.152	0.013	0.107	0.204	
<i>FROE4</i>	0.150	0.150	0.012	0.113	0.198	
<i>FROE5</i>	0.148	0.147	0.012	0.118	0.192	
<i>FROE6</i>	0.147	0.143	0.013	0.122	0.185	
<i>FROE7</i>	0.145	0.139	0.014	0.119	0.179	
<i>FROE8</i>	0.143	0.138	0.016	0.116	0.175	
<i>FROE9</i>	0.141	0.138	0.018	0.112	0.175	
<i>FROE10</i>	0.140	0.138	0.021	0.109	0.175	
<i>FROE11</i>	0.138	0.137	0.023	0.106	0.175	
<i>FROE12</i>	0.136	0.135	0.026	0.102	0.176	

*Notes:* This table presents the descriptive statistics of the state variables used in the VAR (panel A) and analysts' forecasts (panel B) approaches. The sample period for the VAR approach spans for the period 1969:12 – 2014:12 (*i.e.* 541 months); whereas the sample period for the analysts' forecasts approach covers from 1990:01 to 2014:12. For the VAR approach,  $r_M^e$  is the excess market returns, *TY* is the term yield spread, *PE* is the log smoothed *PE* ratio and *VS* is the small-stock value spread. For the analysts' forecasts approach, *DPS* is the dividend per share, *EPS* is the basic earnings per share, *BV* is the book value per share, *ROE* is the returns on common equity, *LTG* is the long-term EPS growth rate, and *FROE1* to *FROE12* denotes the one-year-ahead to twelve-year ahead ROE forecasts. *SD* denotes standard deviation, *Min* is the minimum value, *Max* is the maximum value and  $\rho(1)$  is the first-order autocorrelation. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.



**Table 2****TV-VAR parameter estimates for aggregate stock market returns**

	Constant	$r_{M,t}^e$	$TY_t$	$PE_t$	$VS_t$	$R^2$ (%)
$r_{M,t+1}^e$	0.394*** [0.041]	-0.033 [0.023]	0.001 [0.001]	-0.107*** [0.013]	-0.040** [0.020]	7.90
$TY_{t+1}$	0.989** [0.501]	0.657*** [0.131]	0.901*** [0.008]	-0.564*** [0.168]	0.547*** [0.110]	88.00
$PE_{t+1}$	0.302*** [0.039]	0.435*** [0.012]	0.000 [0.001]	0.915*** [0.010]	-0.027* [0.014]	94.90
$VS_{t+1}$	-0.006 [0.051]	0.056*** [0.018]	0.000 [0.001]	0.057*** [0.018]	0.881*** [0.013]	84.80

*Notes:* This table reports the first-order TV-VAR OLS estimates average across different estimation windows for the period 1969:12 – 2014:12. The associated Newey-West standard errors (with 12 lag) are reported in the square bracket. The state variables used in the TV-VAR model include the excess market returns ( $r_M^e$ ), the term yield spread ( $TY$ ), the 10-year smoothed  $PE$  ratio ( $PE$ ), and small-stock value spread ( $VS$ ). The dependent variable of each regression is presented in the first column and the coefficients of explanatory variables are shown from the second through the sixth columns. The mean adjusted- $R^2$  (in percentage term) is reported in the last column. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.

**Table 3****The attributes of cash flow and discount rate news**

News Cov / Corr	$N_{CF}$	$N_{DR}$
$N_{CF}$	0.0043 (0.0655)	0.0039 (0.8268)
$N_{DR}$	0.0039 (0.8268)	0.0052 (0.0720)
News Functions	$e1'+e1'\lambda$	$e1'\lambda$
$r_M^e$ shocks	0.6188	-0.3812
$TY$ shocks	-0.0465	-0.0465
$PE$ shocks	-0.6636	-0.6636
$VS$ shocks	0.4298	0.4298

*Notes:* This table reports the attributes of the cash flow news ( $N_{CF}$ ) and discount rate news ( $N_{DR}$ ) estimated from the TV-VAR model for the period 1969:12 – 2014:12. The top panel shows the variance-covariance of both news series. The values in the bracket are the correlation matrix of news series with the diagonal elements represent the standard deviations of the news terms. The bottom panel shows the time-series average of the linear function coefficients of  $N_{CF}$  ( $e1'+e1'\lambda$ ) and  $N_{DR}$  ( $e1'\lambda$ ), where  $\lambda = \rho\Gamma(I - \rho\Gamma)^{-1}$ ,  $\Gamma$  is the point estimates of the VAR matrix and  $\rho = 0.95^{1/12}$ .  $r_M^e$  is the excess market returns,  $TY$  is the term yield spread,  $PE$  is the log smoothed  $PE$  ratio and  $VS$  is the small-stock value spread.

**Table 4****Correlations among the four news series**

	$SN_{CF}^{IR}$	$SN_{CF}^R$	$SN_{DR}^{IR}$	$SN_{DR}^R$
$SN_{CF}^{IR}$	1			
$SN_{CF}^R$	0	1		
$SN_{DR}^{IR}$	0.896	0	1	
$SN_{DR}^R$	0	0.825	0	1

*Notes:* This table reports the correlations of the four scaled news series: irrational cash flow news ( $SN_{CF}^{IR}$ ), rational cash flow news ( $SN_{CF}^R$ ), irrational discount rate news ( $SN_{DR}^{IR}$ ), and rational discount rate news ( $SN_{DR}^R$ ).

**Table 5****The stock price movements in respond to four news series computed from TV-VAR**

	Growth	2	3	4	Value
<i>Panel A: Irrational cash flow beta</i>					
Small	0.035 [1.408]	0.029 [1.498]	0.023 [1.516]	0.026* [1.726]	0.020 [1.460]
2	0.040** [1.994]	0.026* [1.692]	0.020 [1.542]	0.023* [1.871]	0.017 [1.194]
3	0.034* [1.955]	0.035** [2.584]	0.021** [2.011]	0.016 [1.429]	0.021* [1.860]
4	0.037* [1.836]	0.024** [2.223]	0.018* [1.872]	0.016 [1.477]	0.019 [1.430]
Large	0.024 [1.616]	0.010 [0.859]	0.008 [0.914]	0.012 [1.250]	0.011 [0.902]
<i>Panel B: Rational cash flow beta</i>					
Small	0.023 [0.152]	0.018 [0.137]	0.007 [0.057]	-0.009 [-0.078]	-0.023 [-0.170]
2	0.085 [0.593]	0.045 [0.337]	0.032 [0.251]	0.046 [0.339]	0.024 [0.156]
3	0.131 [0.941]	0.063 [0.470]	0.075 [0.591]	0.071 [0.524]	0.054 [0.341]
4	0.130 [0.995]	0.111 [0.852]	0.111 [0.816]	0.097 [0.663]	0.099 [0.572]
Large	0.229* [1.955]	0.163 [1.192]	0.174 [1.172]	0.171 [1.102]	0.190 [1.290]
<i>Panel C: Irrational discount rate beta</i>					
Small	-0.003 [-0.145]	-0.008 [-0.479]	-0.003 [-0.206]	-0.007 [-0.525]	0.000 [0.038]
2	-0.009 [-0.498]	-0.004 [-0.345]	0.000 [0.007]	-0.006 [-0.527]	0.000 [-0.037]
3	-0.005 [-0.299]	-0.012 [-0.983]	-0.005 [-0.497]	0.000 [-0.030]	-0.006 [-0.609]
4	-0.011 [-0.688]	-0.001 [-0.139]	-0.001 [-0.133]	-0.001 [-0.135]	0.003 [0.244]
Large	0.000 [-0.012]	0.009 [0.909]	0.011 [1.303]	0.006 [0.686]	0.008 [0.792]
<i>Panel D: Rational discount rate beta</i>					
Small	1.104*** [3.477]	0.944*** [3.457]	0.869*** [3.568]	0.820*** [3.421]	0.882*** [3.468]
2	1.043*** [3.323]	0.916*** [3.292]	0.827*** [3.346]	0.789*** [3.291]	0.916*** [3.365]
3	0.944*** [3.088]	0.870*** [3.338]	0.770*** [3.279]	0.725*** [3.160]	0.850*** [3.382]
4	0.891*** [3.036]	0.818*** [3.181]	0.759*** [3.081]	0.709*** [3.058]	0.839*** [3.141]
Large	0.624** [2.397]	0.673*** [2.931]	0.588*** [2.847]	0.608*** [2.801]	0.649*** [2.801]

*Notes:* This table presents the four betas computed based on the news series retrieved from the time-varying VAR (TV-VAR) approach for portfolio sorted based on size (*ME*) and book-to-market (*BE/ME*) ratio from December 1969 to December 2014 in four panels. Panel A and B report the irrational cash flow beta ( $\beta_{i,CF}^{IR}$ ) and the rational cash flow beta ( $\beta_{i,CF}^R$ ), respectively. Panel C and D show the irrational discount rate beta ( $\beta_{i,DR}^{IR}$ ) and the rational discount rate beta ( $\beta_{i,DR}^R$ ), respectively. The estimation is based on the following regression:

$$r_{i,t} = \alpha + \beta SN_{j,t}^E + \varepsilon_t, \quad SN_{j,t}^E = \{SN_{CF,t}^{IR}, SN_{CF,t}^R, SN_{DR,t}^{IR}, SN_{DR,t}^R\}$$

where  $r_{i,t}$  represents the portfolio returns and  $SN_{j,t}^E$  denotes one of the four scaled news series computed in equations (15) – (18). The  $\beta$  is the beta estimate corresponds to one of the four new series applied in the above regression. Portfolios are sorted based on *BE/ME* ratio from left to right and based on firm size (*ME*) from top to bottom in each panel. “Growth” portfolio has the lowest *BE/ME* ratio, “value” portfolio has the highest *BE/ME* ratio, “small” portfolio has the lowest *ME*, and “large” portfolio has the highest *ME*. HAC standard error is used and the values shown in square bracket are Newey-West *t*-statistics. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.

**Table 6**

**The proportion of the irrational beta relative to the rational beta in CF and DR channels under the TV-VAR approach**

	Growth	2	3	4	Value
<i>Panel A: Proportion of irrational cash flow beta</i>					
Small	0.608	0.616	0.757	1.547	6.075
2	0.323	0.366	0.386	0.331	0.406
3	0.208	0.355	0.214	0.181	0.280
4	0.221	0.176	0.143	0.142	0.161
Large	0.095	0.059	0.044	0.065	0.056
<i>Panel B: Proportion of irrational discount rate beta</i>					
Small	0.003	0.008	0.003	0.008	0.000
2	0.009	0.005	0.000	0.007	0.001
3	0.005	0.014	0.007	0.000	0.007
4	0.013	0.002	0.002	0.002	0.003
Large	0.000	0.013	0.018	0.009	0.013

*Notes:* This table presents the proportion of irrational cash flow beta over the rational cash flow beta  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  in Panel A, and the proportion of irrational discount rate beta over the rational discount rate beta  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$  in Panel B for 25 portfolios sorted based on size (*ME*) and book-to-market (*BE/ME*) ratio. The cash flow and discount rate news are estimated under the TV-VAR specification. The estimation covers the period from December 1969 to December 2014.

**Table 7**  
**The stock price movements in respond to four news series computed from VAR**

	Growth	2	3	4	Value
<i>Panel A: Irrational cash flow beta</i>					
Small	0.023** [2.079]	0.016** [1.990]	0.017** [2.167]	0.017** [2.277]	0.018** [2.373]
2	0.021** [2.287]	0.017** [2.181]	0.017** [2.424]	0.015** [2.118]	0.014* [1.721]
3	0.023*** [2.667]	0.019*** [2.654]	0.014* [1.944]	0.014** [1.984]	0.013* [1.795]
4	0.019** [2.485]	0.021*** [3.009]	0.015** [2.158]	0.013** [1.980]	0.016** [2.053]
Large	0.021*** [3.220]	0.017*** [3.029]	0.015** [2.601]	0.012** [1.988]	0.016** [2.193]
<i>Panel B: Rational cash flow beta</i>					
Small	0.638*** [11.899]	0.551*** [11.251]	0.506*** [9.520]	0.470*** [8.826]	0.490*** [8.298]
2	0.655*** [12.367]	0.562*** [11.563]	0.501*** [10.497]	0.492*** [10.668]	0.545*** [8.605]
3	0.641*** [13.757]	0.553*** [12.236]	0.502*** [12.939]	0.478*** [10.896]	0.514*** [10.617]
4	0.604*** [13.665]	0.550*** [15.174]	0.521*** [11.116]	0.481*** [12.048]	0.567*** [13.274]
Large	0.504*** [11.846]	0.498*** [15.094]	0.452*** [11.789]	0.469*** [9.701]	0.520*** [11.887]
<i>Panel C: Irrational discount rate beta</i>					
Small	0.016** [2.328]	0.010* [1.731]	0.010* [1.798]	0.009* [1.948]	0.009* [1.827]
2	0.015** [2.453]	0.011** [2.117]	0.010** [2.047]	0.008* [1.932]	0.009* [1.729]
3	0.013** [2.399]	0.009** [1.955]	0.008* [1.771]	0.007** [2.030]	0.008* [1.715]
4	0.011** [2.317]	0.008* [1.708]	0.008* [1.930]	0.008* [1.875]	0.011** [2.271]
Large	0.009** [2.102]	0.008* [1.852]	0.007** [1.978]	0.009** [2.447]	0.008** [2.028]
<i>Panel D: Rational discount rate beta</i>					
Small	0.623*** [17.906]	0.520*** [14.796]	0.466*** [12.956]	0.423*** [10.579]	0.455*** [9.671]
2	0.604*** [17.603]	0.513*** [12.287]	0.453*** [12.038]	0.430*** [11.430]	0.490*** [9.723]
3	0.568*** [16.422]	0.485*** [13.302]	0.438*** [12.983]	0.414*** [10.673]	0.479*** [11.023]
4	0.544*** [17.427]	0.490*** [14.577]	0.457*** [12.647]	0.427*** [11.704]	0.476*** [10.739]
Large	0.464*** [14.246]	0.437*** [13.891]	0.400*** [12.833]	0.419*** [8.998]	0.420*** [8.329]

*Notes:* This table presents the four betas computed based on the news series retrieved from the constant VAR approach for portfolio sorted based on size and book-to-market ( $BE/ME$ ) ratio from December 1969 to December 2014 in four panels. Panel A and B report the irrational cash flow beta ( $\beta_{i,CF}^{IR}$ ) and the rational cash flow beta ( $\beta_{i,CF}^R$ ), respectively. Panel C and D show the irrational discount rate beta ( $\beta_{i,DR}^{IR}$ ) and the rational discount rate beta ( $\beta_{i,DR}^R$ ), respectively. The estimation is based on the following regression:

$$r_{i,t} = \alpha + \beta SN_{j,t}^E + \varepsilon_t, \quad SN_{j,t}^E = \{SN_{CF,t}^{IR}, SN_{CF,t}^R, SN_{DR,t}^{IR}, SN_{DR,t}^R\}$$

where  $r_{i,t}$  represents the portfolio returns and  $SN_{j,t}^E$  denotes one of the four scaled news series computed in equations (15) – (18). The  $\beta$  is the beta estimate corresponds to one of the four new series applied in the above regression. Portfolios are sorted based on  $BE/ME$  ratio from left to right and based on firm size ( $ME$ ) from top to bottom in each panel. “Growth” portfolio has the lowest  $BE/ME$  ratio, “value” portfolio has the highest  $BE/ME$  ratio, “small” portfolio has the lowest  $ME$ , and “large” portfolio has the highest  $ME$ . HAC standard error is used and the values shown in square bracket are Newey-West  $t$ -statistics. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.

**Table 8****The proportion of irrational beta relative to rational beta in CF and DR channels under the VAR approach**

	Growth	2	3	4	Value
<i>Panel A: Proportion of irrational cash flow beta</i>					
Small	0.035	0.029	0.033	0.034	0.036
2	0.032	0.030	0.033	0.029	0.024
3	0.035	0.033	0.026	0.028	0.024
4	0.030	0.037	0.027	0.026	0.028
Large	0.040	0.034	0.033	0.025	0.030
<i>Panel B: Proportion of irrational discount rate beta</i>					
Small	0.025	0.020	0.020	0.021	0.020
2	0.025	0.021	0.021	0.019	0.017
3	0.022	0.019	0.018	0.017	0.016
4	0.020	0.016	0.017	0.018	0.022
Large	0.018	0.017	0.018	0.021	0.019

*Notes:* This table presents the proportion of irrational cash flow beta over the rational cash flow beta  $\beta_{i,CF}^{IR} / \beta_{i,CF}^R$  in Panel A, and the proportion of irrational discount rate beta over the rational discount rate beta  $\beta_{i,DR}^{IR} / \beta_{i,DR}^R$  in Panel B for 25 portfolios sorted based on size (*ME*) and book-to-market (*BE/ME*) ratio. The cash flow and discount rate news are estimated under the VAR specification. The estimation covers the period from December 1969 to December 2014.

**Table 9**  
**The stock price movements in respond to four news series computed from AF**

	Growth	2	3	4	Value
Panel A: Irrational cash flow beta					
Small	-0.007 [-0.152]	-0.003 [-0.084]	-0.026 [-0.844]	-0.023 [-0.703]	-0.041 [-1.192]
2	0.014 [0.393]	-0.003 [-0.094]	-0.024 [-0.845]	-0.014 [-0.487]	-0.009 [-0.241]
3	0.007 [0.227]	-0.009 [-0.303]	-0.016 [-0.617]	-0.023 [-0.769]	-0.007 [-0.197]
4	0.029 [0.849]	-0.017 [-0.672]	-0.016 [-0.548]	-0.009 [-0.318]	0.004 [0.102]
Large	0.012 [0.588]	-0.004 [-0.174]	-0.019 [-0.772]	0.005 [0.143]	0.002 [0.061]
Panel B: Rational cash flow beta					
Small	0.242* [1.944]	0.206*** [2.707]	0.210*** [3.390]	0.188** [2.583]	0.209*** [2.726]
2	0.238** [2.376]	0.183*** [2.638]	0.176*** [2.974]	0.241*** [3.681]	0.278*** [2.860]
3	0.223*** [2.764]	0.200*** [2.916]	0.191*** [3.251]	0.158** [2.275]	0.146* [1.820]
4	0.182** [2.034]	0.157** [2.446]	0.190*** [2.649]	0.173** [2.202]	0.280*** [2.681]
Large	0.199* [1.924]	0.193*** [2.669]	0.212*** [3.436]	0.275*** [3.352]	0.332** [2.601]
Panel C: Irrational discount rate beta					
Small	0.063 [1.409]	0.044 [1.164]	0.064** [2.136]	0.059* [1.894]	0.087*** [2.622]
2	0.038 [1.023]	0.046 [1.312]	0.061** [2.071]	0.048 [1.594]	0.05 [1.228]
3	0.048 [1.386]	0.049 [1.468]	0.05 [1.495]	0.058 [1.640]	0.049 [1.381]
4	0.017 [0.560]	0.056* [1.946]	0.054 [1.645]	0.042 [1.224]	0.042 [1.120]
Large	0.02 [0.772]	0.034 [1.227]	0.037 [1.258]	0.026 [0.739]	0.041 [0.951]
Panel D: Rational discount rate beta					
Small	0.967*** [4.577]	0.804*** [4.488]	0.706*** [3.953]	0.653*** [4.089]	0.696*** [3.940]
2	0.937*** [4.743]	0.805*** [4.299]	0.708*** [4.106]	0.654*** [4.158]	0.767*** [4.147]
3	0.906*** [4.773]	0.796*** [4.254]	0.718*** [4.279]	0.734*** [4.548]	0.855*** [4.983]
4	0.913*** [4.796]	0.807*** [4.682]	0.784*** [4.682]	0.740*** [4.741]	0.779*** [4.138]
Large	0.729*** [5.247]	0.687*** [5.467]	0.613*** [4.457]	0.681*** [3.988]	0.729*** [3.791]

*Notes:* This table presents the four betas computed based on the news series retrieved from the constant analysts' forecast (AF) approach for portfolio sorted based on size and book-to-market ( $BE/ME$ ) ratio from January 1990 to December 2014 in four panels. Panel A and B report the irrational cash flow beta ( $\beta_{i,CF}^{IR}$ ) and the rational cash flow beta ( $\beta_{i,CF}^R$ ), respectively.

Panel C and D show the irrational discount rate beta ( $\beta_{i,DR}^{IR}$ ) and the rational discount rate beta ( $\beta_{i,DR}^R$ ), respectively. The estimation is based on the following regression:

$$r_{i,t} = \alpha + \beta SN_{j,t}^E + \varepsilon_t, \quad SN_{j,t}^E = \{SN_{CF,t}^{IR}, SN_{CF,t}^R, SN_{DR,t}^{IR}, SN_{DR,t}^R\}$$

where  $r_{i,t}$  represents the portfolio returns and  $SN_{j,t}^E$  denotes one of the four scaled news series computed in equations (15) – (18). The  $\beta$  is the beta estimate corresponds to one of the four new series applied in the above regression. Portfolios are sorted based on  $BE/ME$  ratio from left to right and based on firm size ( $ME$ ) from top to bottom in each panel. “Growth” portfolio has the lowest  $BE/ME$  ratio, “value” portfolio has the highest  $BE/ME$  ratio, “small” portfolio has the lowest  $ME$ , and “large” portfolio has the highest  $ME$ . HAC standard error is used and the values shown in square bracket are Newey-West  $t$ -statistics. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.

**Table 10**  
**Prices of risks**

	CAPM (1)	Two-beta model (2)	Four-beta model		
			TV-VAR (3)	VAR (4)	AF (5)
$\lambda_M$	0.006*** [2.823]				
$\lambda_{CF}$		-0.002 [-0.272]			
$\lambda_{DR}$		0.009*** [2.715]			
$\lambda_{CF}^{IR}$			-0.622*** [-3.672]	-0.282** [-2.037]	-0.183 [-1.438]
$\lambda_{CF}^R$			0.015* [1.859]	0.037* [1.807]	0.002 [0.197]
$\lambda_{DR}^{IR}$			-0.554*** [-4.176]	-1.017*** [-3.833]	-0.086 [-0.745]
$\lambda_{DR}^R$			0.023*** [4.723]	0.004 [0.203]	0.013*** [3.123]
$R^2$	0.003	0.194	0.378	0.196	0.235
$RMSPE$	0.024	0.021	0.016	0.019	0.021
$MPE$ (%)	0.023	0.023	0.009	0.009	0.007

*Notes:* This table presents the results of the Fama-Macbeth regression for the Capital Asset Pricing Model (CAPM), the CV's two-beta model, and the four-beta models computed with the news series retrieved from the time-varying VAR (TV-VAR), the constant VAR and the analysts' forecast (AF) approach. The test assets are 25 portfolios sorted based on size (*ME*) and book-to-market (*BE/ME*) ratio. The risk premium estimates are the time-series average of the cross-sectional parameter estimates for the period of 1969:12 – 2014:12, except the four-beta model (AF), which has the sample period of 1990:1 – 2014:12.  $\lambda_M$  is the price of market risk,  $\lambda_{CF}$  is the price of cash flow risk,  $\lambda_{DR}$  is the price of discount rate risk,  $\lambda_{CF}^{IR}$  is the price of irrational cash flow risk,  $\lambda_{CF}^R$  is the price of rational cash flow risk,  $\lambda_{DR}^{IR}$  is the price of irrational discount rate risk, and  $\lambda_{DR}^R$  is the price of rational discount rate risk. The heteroskedastic and autocorrelation consistent *t*-statistics are presented in the square bracket. The adjusted- $R^2$  ( $R^2$ ) statistic, the root-mean-squared-pricing-errors (*RMSPE*), the mean-pricing-errors (*MPE*) are presented in the last three rows. \*, \*\* and \*\*\* indicate statistical significance at 10%, 5% and 1% level, respectively.



**Table 11: Anomalies test performance**

Strategy	$\mu$	$\sigma$	$\beta_{CF}^{IR}$	$\beta_{CF}^R$	$\beta_{DR}^{IR}$	$\beta_{DR}^R$	$\alpha_{CAPM}$	$\alpha_{2B}$	$\alpha_{4B}$
RMRF	0.521	4.597	0.020	0.154	0.003	0.702	-0.067	0.024	0.182
SMB	0.148	3.136	0.013	-0.128	-0.012	0.221	0.084	-0.139	-0.102
HML	0.389	2.928	-0.013	-0.053	0.004	-0.066	0.483	0.390	0.044
RMW	0.292	2.295	-0.002	0.002	0.002	-0.084	0.347	0.367	0.490
CMA	0.370	2.017	-0.018	-0.044	0.005	-0.078	0.468	0.384	-0.226
UMD	0.667	4.395	0.007	-0.053	-0.012	-0.056	0.756	0.690	0.535
STR	0.470	3.281	0.010	0.079	-0.006	0.096	0.348	0.463	0.321
LTR	0.291	2.577	-0.006	-0.045	-0.004	0.073	0.301	0.186	-0.467
BETA	0.008	6.699	-0.046	-0.053	0.020	-0.697	0.544	0.542	0.082
ACC	0.366	2.761	-0.003	-0.009	0.006	-0.056	0.410	0.402	0.775
NI	0.528	3.263	-0.017	0.007	0.009	-0.175	0.659	0.671	0.428
VAR	0.669	8.046	-0.057	0.044	0.020	-0.830	1.232	1.396	0.128
RESVAR	0.802	7.341	-0.048	0.130	0.018	-0.726	1.242	1.516	0.331

*Notes:* This table presents the performance of capital asset pricing model (CAPM), the two-beta model (2B), and the four-beta model (4B) in pricing the anomalies.  $\alpha$  denotes the abnormal returns of anomaly computed as the difference between the mean excess returns ( $\mu$ ) and the predicted excess returns computed using different asset pricing models. The test covers the period from 1969:12 to 2014:12. All data are expressed in percentage term except beta estimates.

**Table 12: Mean absolute alpha of asset pricing models**

Strategy	$\bar{\alpha}_{CAPM}$ (%)	$\bar{\alpha}_{2B}$ (%)	$\bar{\alpha}_{4B}$ (%)
All (not scaled)	0.534	0.552	0.316
All (scaled)	0.134	0.132	0.100
FF-3 (not scaled)	0.211	0.185	0.109
FF-3 (scaled)	0.069	0.061	0.029
FFC-4 (not scaled)	0.347	0.311	0.216
FFC-4 (scaled)	0.095	0.085	0.052
FF-5 (not scaled)	0.289	0.261	0.209
FF-5 (scaled)	0.118	0.107	0.083

*Notes:* This table report the mean absolute alpha,  $\bar{\alpha}$ , of the capital asset pricing model (CAPM), the two-beta model (2B), and the four-beta model (4B) averaged across all anomaly strategies, three-factor, four-factor and five-factor anomalies. The scaled mean absolute alpha is computed as  $\alpha_i \times \sigma_{RMRF} / \sigma_i$ , where the alpha,  $\alpha_i$ , and the volatility,  $\sigma_i$ , of anomaly portfolio are obtained from Table 11.